Topological Ramsey spaces in creature forcing

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Pigeonhole principles play a key role in many aspects of creature forcing. Rosłanowski and Shelah proved a general scheme of Ramsey-type results for partitions of countable sets of finite functions, utilizing and extending Glazer's method of proof of Hindman's Theorem to collections of pure candidates for a large class of creature forcings, proving the existence of idempotent ultrafilters with respect to the related operation. As a consequence, they obtained ultrafilters on countable base set of finite functions which are generated by pure candidates, analogously to ultrafilters generated by infinite block sequences using Hindman's Theorem.

For three specific examples given by Roslanowski and Shelah, we construct topological Ramsey spaces which are dense, hence forcing equivalent, in the collections of pure candidates. This means these spaces satisfy Ellentuck's Theorem in the related exponential topology: Every subset with the property of Baire has the Ramsey property. As a consequence, we recover Rosłanowski and Shelah's result on partition relations for colorings on countable sets of finite functions for these examples. In order to prove that the pigeonhole principle (Axiom A.4) holds for two of the examples, we prove an augmented version of the Product Tree Theorem of Di Prisco, Llopis and Todorcevic.

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