Generic norms and metrics on countable Abelian groups

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For a countable Abelian group G we investigate the Polish space of all norms, resp. invariant metrics, on G. We are mainly interested in generic properties of the space, i.e. properties that are satisfied by all the norms/metrics on G except those coming from a set of first category. We prove that for every countable Abelian group G that is unbounded, i.e. it has elements of arbitrarily high order, there is a dense set of norms on G with which it is isometric to the rational Urysohn space, and a comeager set of norms such that the completion is isometric to the Urysohn space. That generalizes results of Cameron, Vershik, Niemiec and others.

Then we prove that for every G such that $G \cong \bigoplus_{\mathbb{N}} G$ there is a comeager set of norms on G such that all of them give rise to the same metric group after completion. If moreover G is unbounded, then using a result of Melleray and Tsankov we get that the completion is extremely amenable. Among our corollaries is the result that all the known universal Abelian Polish groups, e.g. the Shkarin's group, are extremely amenable.