PFA(S) implies there are many S-names

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PFA(S) is the statement that there is a coherent Souslin tree S and that Martin's Axiom holds for the class of proper S-preserving posets. The weaker statement SA_{ω_1} for S-preserving ccc posets was introduced by Larson and Todorcevic to solve Katetov's problem. Todorcevic introduced PFA(S) and showed its consistency from a supercompact cardinal. Todorcevic, Larson-Tall, and Tall have proven many consequences of PFA(S)[S] – i.e. statements holding in the forcing extension by S. We continue the investigation.

We cite two examples that are joint results with F. Tall.

Theorem *PFA(S) implies that forcing with S produces a model in which perfect sequential preimages of ω_1 contain a copy of ω_1.*

Another example, using the stronger principle MM(S) and using very strong stationary set reflection, is

Theorem *MM(S) implies that after forcing with S locally compact normal spaces are $\aleph_1$-CWH.*

Question *Can the hypotheses be weakened to PFA(S)?*

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