Topological entropy on totally disconnected locally compact groups

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We study the topological entropy h_{top} for continuous endomorphisms $\phi : G \to G$ of totally disconnected locally compact groups G. In this setting, we prove the Addition Theorem for h_{top} under suitable assumptions, that is, if H is a closed ϕ-invariant subgroup of G and H is either compact or normal in G, then

$$h_{top}(\phi) = h_{top}(\phi \restriction_H) + h_{top}(\bar{\phi}),$$

whenever $\phi \restriction_H$ is surjective and the endomorphism $\bar{\phi} : G/H \to G/H$ induced by ϕ is injective.

As an application we give a dynamical interpretation of the scale $s(\phi)$, by showing that $\log s(\phi)$ is the topological entropy of a suitable map induced by ϕ. We find also necessary and sufficient conditions for the equality $\log s(\phi) = h_{top}(\phi)$ to hold.