Measuring noncompactness and discontinuity

Ondřej F. K. Kalenda[1]

kalenda@karlin.mff.cuni.cz

There are several ways how to measure (relative) noncompactness of sets and operators in Banach spaces. I will survey and compare measures of noncompactness in several topologies. Further, there is a close connection between various types of compactness in Banach spaces with certain types of continuity. For example, the Arzelà–Ascoli theorem reveals a relation of continuity and norm-compactness and some Grothendieck's theorems show connections between weak compactness and Mackey continuity and between Mackey compactness and weak sequential continuity. I will address a quantitative approach to these relationships, i.e., possible strengthenings of certain implications and equivalences to suitable inequalities. The lecture will be based mainly on the papers [1],[2],[3],[4].

[1] B. Cascales, O. F. K. Kalenda, and J. Spurný, *A quantitative version of James's compactness theorem*, Proc. Edinb. Math. Soc. (2) **55** (2012), no. 2, 369–386

[2] M. Kačena, O. F. K. Kalenda, and J. Spurný, *Quantitative Dunford-Pettis property*, Adv. Math. **234** (2013), 488–527

[3] O. F. K. Kalenda and J. Spurný, *Quantification of the reciprocal Dunford-Pettis property*, Studia Math. **210** (2012), no. 3, 261–278

[4] O. F. K. Kalenda and J. Spurný, *On quantitative Schur and Dunford-Pettis properties*, Bull. Aust. Math. Soc. **91** (2015), no. 3, 471–486

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