On dimension of inverse limits with set-valued functions

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Recently, several topological properties of inverse limits of compacta with upper semi-continuous set-valued functions have been studied by many authors. The study of such inverse limits has developed into one of rich topics of geometric topology. There are many differences between the theory of inverse limits with mappings and the theory with set-valued functions. In this paper, we investigate the dimension of inverse limits with set-valued functions. To evaluate the dimension of the inverse limit $\varprojlim \{X_i, f_{i,i+1}\}$ of given inverse sequence $\{X_i, f_{i,i+1}\}_{i=1}^{\infty}$ of compacta with set-valued functions satisfying

$$\dim\{x \in X_{i+1} \mid \dim f_{i,i+1}(x) \geq 1\} \leq 0 \ (i \in \mathbb{N}),$$

we define expand-contract sequences in $\{X_i, f_{i,i+1}\}_{i=1}^{\infty}$ and an index $\tilde{J}(\{X_i, f_{i,i+1}\})$. By use of the index, we prove that

$$\dim \varprojlim \{X_i, f_{i,i+1}\} \leq \tilde{J}(\{X_i, f_{i,i+1}\}) + \sup\{\dim X_i \mid i \in \mathbb{N}\}.$$

Moreover, we evaluate lower bounds of dimensions of some inverse limits of 1-dimensional compacta with set-valued functions.

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