Random elements of large groups – Continuous case

Udayan B. Darji, Márton Elekes, Kende Kalina, Viktor Kiss, Zoltán Vidnyánszky*

kivi@cs.elte.hu

We investigate the behavior of random elements of some non-locally--compact groups. Since such groups do not carry a natural measure, we use the following notion introduced by Christensen.

Definition. Christensen Let G be a Polish group. A subset $H \subset G$ is called *Haar null* if there exists a Borel probability measure μ on G and a Borel set $B \supset H$ such that for every $g, h \in G$, $\mu(gBh) = 0$.

It is well-known that the family of Haar null sets form a σ-ideal and that they coincide with the family of measure zero sets (with respect to a left (or equivalently, a right) Haar measure) if G is locally compact.

We examine the group of increasing homeomorphisms of the unit interval and the unit circle. We describe the behavior of the random element of these groups by characterizing the Haar positive (that is, not Haar null) conjugacy classes of these groups. We obtain as a corollary of this characterization that the group of increasing homeomorphisms of the unit interval can be written as a union of a meager and a Haar null set.

We also examine the group of unitary transformations of the separable Hilbert space. We give a partial description of Haar positive conjugacy classes by finding a countable set of conjugacy classes and proving that every conjugacy class not contained in it is Haar null.