Generic objects in topology

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We shall present a new, category-theoretic, concept of a *generic object* in various subcategories of the category of compact Hausdorff spaces. Roughly speaking, a topological space U is *generic* over a class (category) of topological spaces if one of the players has a winning strategy in a natural infinite game producing an inverse sequence converging to U. For example, consider the unit interval $[0,1]$ and assume that two players alternately choose continuous surjections $[0,1] \leftarrow [0,1]$. After infinitely many steps, an inverse sequence of continuous surjections of the unit interval is produced. One can look at its inverse limit as the result of the play. It turns out that each of the players has a strategy leading to the pseudo-arc, a rather special and well-known chainable continuum. By this way, the pseudo-arc is generic in the category of continuous surjections of the unit interval.

We shall present a general framework capturing objects as above. As a byproduct, we present a new proof of the homogeneity of spaces like the Cantor set and generalized Baire spaces.

Theorem *Given a cardinal κ, every homeomorphism between closed nowhere dense subsets of κ^ω extends to an auto-homeomorphism of κ^ω.*