Notes on free (Abelian) topological groups

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In this talk, I present some results on k-space and Fréchet–Uryshon property of free topological groups and free Abelian topological groups. Some questions are posed.

Theorem *Let X be a topological space in which the closure of a bounded subset in X is compact. If $F_5(X)$ is Fréchet–Urysohn, then X is compact or discrete.*

Theorem *Let X be a non-metrizable, Lašnev space. Then the following are equivalent.*

1. *$A(X)$ is a k-space.*
2. *$A_n(X)$ is a k-space for each n.*
3. *$A_4(X)$ is a k-space.*
4. *X is a topological sum of a k-space with a countable k-network consisting of compact subsets and a discrete space.*

Theorem *Assume $\mathfrak{b} = \omega_1$. For a non-metrizable Lašnev spaces X, $A_3(X)$ is a k-space if and only if $A(X)$ is a k-space.*

Theorem *Assume $\mathfrak{b} > \omega_1$. There exists a non-metrizable Lašnev space X such that $A_3(X)$ is a k-space but $A(X)$ is not.*