The Ramsey property for Banach spaces, Choquet simplices, and their noncommutative analogs

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We compute the *universal minimal flows* of the automorphism groups of structures coming from functional analysis and convex: the Gurarij space $\mathbb{G}$, the Poulsen simplex $\mathbb{P}$, and their noncommutative analogs. The Gurarij space is the unique separable approximately ultrahomogeneous Banach space that contains ℓ_n^∞ for every $n \in \mathbb{N}$, while $\mathbb{P}$ is the unique metrizable Choquet simplex with dense extreme boundary. The group Aut $(\mathbb{G})$ of surjective linear isometries of $\mathbb{G}$ is shown to be *extremely amenable*, by proving the *approximate Ramsey property* of the class of finite-dimensional Banach spaces. Similarly the stabilizer $\text{Aut}_p (\mathbb{P})$ of an extreme point p of $\mathbb{P}$ is proven to be extremely amenable, by establishing the approximate Ramsey property of the class of Choquet simplices with a distinguished point. It is then deduced that the universal minimal flow of Aut $(\mathbb{P})$ is $\mathbb{P}$ itself. More generally, we prove that for any closed face F of $\mathbb{P}$, the pointwise stabilizer $\text{Aut}_F (\mathbb{P})$ is *extremely amenable*.

We also provide the natural noncommutative analogs of the results above, formulated in the categories of operator spaces and operator systems. In particular we study the *noncommutative Gurarij space* and the *noncommutative Gurarij system*.

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