Normal spanning trees in uncountable graphs, and almost disjoint families

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In a paper from 2001 (JLMS), Diestel and Leader characterised uncountable graphs that admit a normal spanning tree through a class of forbidden graph-theoretic substructures. These substructures are strongly related to Aronszajn trees, almost disjoint families and ultrafilters.

In this talk we investigate under which circumstances these forbidden substructures can be made nice. We will see that there is a nice solution to this problem under Martin's Axiom+$\neg$CH. However, under CH or $\diamondsuit$, it seems as if we have to allow for many inequivalent forbidden substructures—but so far we only know two inequivalent classes through the work of Diestel and Leader.

I will describe the problems that are involved, draw some connections to topology and the space of ultrafilters $\beta\omega$, and explore parallels to the work of Roitman and Soukup (Fundamenta, 1998) on certain structural properties of almost disjoint families.

This is work in progress, joint with Nathan Bowler and Stefan Geschke.