On lineability of classes of functions with various degrees of (dis)continuity

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We will provide a short overview of recent research on the existence of "large" algebraic structures (e.g., vector spaces, groups) within various classes of real functions. Our focus will be on the classes of functions with various degrees of continuity (discontinuity): almost continuous functions (AC), Darboux functions, Sierpiński–Zygmund functions (SZ). We will prove the following theorem.

Theorem *Assuming the Continuum Hypothesis, the intersection of the classes of almost continuous functions* (AC) *and Sierpiński-Zygmund functions* (SZ) *contains an additive semigroup of size* $2^{\mathfrak{c}}$.

In addition, we will show that it is consistent with ZFC that the cardinality of the "largest" vector space within the class of Sierpiński–Zygmund functions (SZ) is equal to the cardinality of the "largest" vector space within the class of almost continuous Sierpiński–Zygmund functions (AC ∩ SZ). Namely, we show the following result.

Theorem *It is consistent with ZFC that* $\mathcal{L}(\mathrm{AC}\cap\mathrm{SZ}) = \mathcal{L}(\mathrm{SZ}) = (2^{\mathfrak{c}})^+$. (*where* $\mathcal{L}(F)$ *is called the lineability of* $F \subseteq \mathbb{R}^{\mathbb{R}}$ *and is defined as* $\min\{\kappa : F \cup \{0\}$ *doesn't contain a vector space of dim* $\kappa\}$)

It is unclear if the two cardinal numbers from the above theorem can be different under the assumption of the existence of an almost continuous Sierpiński-Zygmund function.

Question *Is it consistent with ZFC that* $\mathrm{AC} \cap \mathrm{SZ} \neq \emptyset$ *and* $\mathcal{L}(\mathrm{AC} \cap \mathrm{SZ}) < \mathcal{L}(\mathrm{SZ})$?