# Recovering a Compact Hausdorff Space $X$ from the Compatibility Ordering on $C(X)$

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Let $X$ and $Y$ be compact Hausdorff spaces. Let $f, g \in C(X)$ where $C(X)$ denotes the space of continuous functions on $X$. We say that $g$ dominates $f$ in the compatibility ordering if $g$ coincides with $f$ on the support of $f$. Our main result states that $X$ and $Y$ are homeomorphic if and only if there exists a compatibility isomorphism $T : C(X) \to C(Y)$. We derive several classical theorems of functional analysis as easy corollaries to our result:

If $X$ and $Y$ are compact Hausdorff spaces, we obtain that they are homeomorphic provided that there exists a bijection $T : C(X) \to C(Y)$ satisfying one of the following conditions:

1. $T$ is a ring isomorphism (Gelfand–Kolmogorov);
2. $T$ is multiplicative (Milgram);
3. $T$ the ordinary pointwise ordering (Kaplansky);
4. $Tf \cdot Tg = 0$ whenever $f \cdot g = 0$ (Jarosz).



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