Selectively sequentially pseudocompact group topologies on Abelian groups

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We call a space X *selectively sequentially pseudocompact* if for each countable family $\{U_n : n \in \mathbb{N}\}$ of non-empty open subsets of X one can choose a point $x_n \in U_n$ for every $n \in \mathbb{N}$ in such a way that the sequence $\{x_n : n \in \mathbb{N}\}$ has a convergent subsequence. Clearly, sequentially compact $\rightarrow$ selectively sequentially pseudocompact $\rightarrow$ strong pseudocompact $\rightarrow$ pseudocompact. Unlike the strong pseudocompactness property of S. García-Ferreira and Y. F. Ortiz-Castillo whose preservation under products even in topological groups remains an open question, the class of selectively sequentially pseudocompact spaces is closed under taking arbitrary products.

We study the following general question: *If an Abelian group admits a pseudocompact group topology, does it also admit a selectively sequentially pseudocompact group topology?* Under the Singular Cardinal Hypothesis (SCH), we provide a positive answer to this question for the following classes of Abelian groups:

1. torsion groups;
2. torsion-free groups;
3. $\mathcal{U}$-free groups in any variety $\mathcal{U}$ of Abelian groups.

This provides a partial answer to a question of S. García-Ferreira and A. H. Tomita (with strong pseudocompactness strengthened to selective sequential compactness).