On Boolean topological groups

Ol'ga V. Sipacheva[1]

o-sipa@yandex.ru

Boolean topological groups often arise in topological algebra as examples of topological groups with exotic properties. They play the key role in the theory of extremally disconnected topological group, because, by Malykhin's 1975 theorem, any extremally disconnected group contains an open Boolean subgroup. The talk considers properties of such groups with special emphasis on free Boolean topological groups and extremally disconnected groups.

Some properties of free Boolean topological groups differ drastically from those of free and free Abelian topological groups. Thus, we show that the free Boolean topological group $B(X)$ on a space X does not always contain X^2 as a subspace (while the free and free Abelian groups on X contain all X^n as closed subspaces) and discuss other examples of such properties.

As mentioned above, Boolean topological groups play the key role in Arhangel'skii's 1967 problem on the existence of extremally disconnected topological groups. So far, only partial results have been obtained. One of results presented in the report is that any closed linearly independent subset of a countable extremally disconnected group contains at most one nonisolated point.

Finally, a natural relationship between countable Boolean topological groups and notions of forcing (in particular, Mathias and Laver forcings) is considered.