Chaos in hyperspaces of nonautonomous discrete systems

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Given a topological space X, let $f_n : X \to X$ be a continuous function for each positive integer n, and $f_\infty = (f_1, f_2, ..., f_n, ...)$. The pair (X, f_∞) denotes the *nonautonomous discrete dynamical system* (NDS, for short). Given a NDS (X, f_∞), it induces a NDS $(\mathcal{K}(X), \overline{f_\infty})$, where $\mathcal{K}(X)$ is the hyperspace of all non-empty compact subsets of X endowed with the Vietoris topology and, for every positive integer n, $\overline{f_n} : \mathcal{K}(X) \to \mathcal{K}(X)$ is the continuous function induced by f_n. We study the interaction of some dynamical properties (like transitivity, weakly mixing, points with dense orbit and density of periodic points) of a NDS (X, f_∞) and its induced NDS $(\mathcal{K}(X), \overline{f_\infty})$. Among other results, we show that $(\mathbb{I}, f_\infty)$ is weakly mixing of order 3 if and only if $(\mathcal{K}(\mathbb{I}), \overline{f_\infty})$ is weakly mixing of order 3, where $\mathbb{I} = [0, 1]$. We also present examples of NDS showing that the classical result stating that transitivity is a sufficient condition for an autonomous discrete dynamical system on the interval to be Devaney chaotic fails to be true for NDS.

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