Base Tree Phenomenological Horizons

Peter Vojtas[1]

vojtas@ksi.mff.cuni.cz

Various mathematical asymptotics (like small/large, slow/fast) create phenomena of horizons we are challenged to perceive.

One such phenomenon can be observed when considering border between absolute convergence and divergence in c_0^+ ([1]). Classical tests for absolute convergence/divergence decide only sets of first category in ℓ^1 , $\ell^2 \setminus \ell^1$ resp. There is an (ω_1, ω_1^*) Hausdoff gap filling this. Comparison ordering of absolutely convergent series is upwards directed, comparison ordering of absolutely divergent series (downwards) has base tree property ([2]) and various cardinal characteristics of bottom and upper part of this horizon seem to depend on axioms/models of ZFC (e.g. equal under CH). Similar phenomenon can be observed on partitions of omega with refinement, see [3].

We show that $\mathfrak{h}\left(\bigcap_{n=1}^{\infty} \ell^{1+\frac{1}{n}} \setminus \ell^1, <^*\right)$ and $\mathfrak{h}(P(\omega)/_{fin}, \subseteq^*)$ are consistently different (see [1]). So, this horizon is behind ℓ^p hierarchy, is topologically large and set-theoretically sensitive.

We present a more general framework for (separative quotients of) horizons, recall some old and new examples, results and problems.

[1] S. Fuchino, H. Mildenberger, S. Shelah, and P. Vojtas, *On absolutely divergent series*, Fund. Math. **160** (1999), no. 3, 255–268

[2] B. Balcar, M. Doucha, and M. Hrusak, *Base tree property*, Order **32** (2015), no. 1, 69–81

[3] S. Krajci, Math. Logic Quaterly **45** (1999), no. 3, 415–420

[1] Initiated while AvHumbold Stiftung Fellow, supported by Czech project P-46