The comparission of topologies for the fundamental group and for generalized covering spaces

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The first traces of attempts to consider the fundamental group as an object, which apart from its algebraic structure also has a topological structure, go back to the fifties, but they remained essentially without echo, till Biss in 2002 published a paper, where he revitalized this idea. The topology proposed by him was different from this inverse-limit topolopgy that had appeared in the fifties already, but although his paper contained several mistakes it must be regaraded the essential paper which revitalized this idea. Amongst the mistakes were the confusion between the fundamental and the first Čech-homotopy group of Hawaiian Earrings, and the assertion that the topology that he had proposed for the fundamental group would always induce the structure of a topological group. The assertion was believed for a couple of years, before almost simultaneously it was disproven by Fabel and Brazas. For the time being, we are aware of 5 different definitions of topologies on the fundamental group. The main part of the talk will be used for desribing the concepts and results on comparing the types of open sets that they generate. The talk will also report on similar results and attempts to extend the definition to those generalized covering spaces which are modelled on the point set $\widetilde{X} := \{w : [0,1] \to X, w(0) = x_0\}/\sim$, i.e. on a set of homotopy classes of paths which just differs from the fundamental group by not requiring that the paths return to the base point.