On completeness and topologizability of countable semigroups

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In this talk we discuss a connection between categorical closedness and topologizability of semigroups. For a class $\mathsf{T}_1\mathsf{S}$ of T_1 topological semigroups we show that a countable semigroup X with finite-to-one shifts is injectively $\mathsf{T}_1\mathsf{S}$-closed if and only if X is $\mathsf{T}_1\mathsf{S}$-nontopologizable in the sense that every T_1 semigroup topology on X is discrete. Moreover, a countable cancellative semigroup X is absolutely $\mathsf{T}_1\mathsf{S}$-closed if and only if every homomorphic image of X is $\mathsf{T}_1\mathsf{S}$-nontopologizable. Also, we discuss a notion of a polybounded semigroup. It is proved that a countable semigroup X with finite-to-one shifts is polybounded if and only if X is $\mathsf{T}_1\mathsf{S}$-closed if and only if X is $\mathsf{T}_z\mathsf{S}$-closed, where $\mathsf{T}_z\mathsf{S}$ is a class of Tychonoff zero-dimensional topological semigroups. We show that polyboundedness provides an automatic continuity of the inversion in T_1 paratopological groups and prove that every cancellative polybounded semigroup is a group.

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