Hereditarily indecomposable continua as Fraïssé limits

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In 2006, Irwin and Solecki introduced projective Fraïssé theory of topological structures and showed that a pre-space of the pseudo-arc is the Fraïssé limit of the class of all finite linear graphs and quotient maps. They also characterized the pseudo-arc as the unique arc-like continuum $\mathbb{P}$ such that for every arc-like continuum Y, every $\varepsilon > 0$, and every continuous surjections $f, g \colon \mathbb{P} \to Y$ there is a homeomorphism $h \colon \mathbb{P} \to \mathbb{P}$ such that $\sup_{x \in \mathbb{P}} d(f(x), g(h(x))) < \varepsilon$.

We consider an approximate framework for Fraïssé theory where the pseudo-arc itself is the Fraïssé limit of the category $\mathcal{I}$ of all continuous surjections of the unit interval, in the category $\sigma\mathcal{I}$ of all arc-like continua and all continuous surjections. The characterizing condition above becomes the *projective homogeneity* condition in our framework.

Similarly, we may consider the category $\mathcal{S}$ of all continuous surjections of the unit circle, and the category $\sigma\mathcal{S}$ of all circle-like continua and all continuous surjections. It turns out there is no Fraïssé limit of $\mathcal{S}$ in $\sigma\mathcal{S}$. However, if we restrict to the subcategory $\mathcal{S}_P \subseteq \mathcal{S}$ of the maps whose degree uses only primes from a fixed set P, and the subcategory $\sigma\mathcal{S}_P \subseteq \sigma\mathcal{S}$ of circle-like continua that are limits of inverse sequences of $\mathcal{S}_P$-maps, with maps that can be approximated by $\mathcal{S}_P$-maps as morphisms, then the corresponding Fraïssé limit is the P-adic pseudo-solenoid $\mathbb{P}_P$, and it is characterized as the unique $\sigma\mathcal{S}_P$-object that is *projectively homogeneous*, or equivalently has the *projective extension property.*

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