Scattered P-spaces of weight ω_1

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Following M. Fréchet, if a space X can be embedded into a space Y, then we write $X \subset_h Y$. If $X \subset_h Y$ and $Y \subset_h X$, then we say that X and Y have the same *topological rank* (K. Kuratowski) or *dimensional type* (W. Sierpiński). Such relations are used in, for example, considering topological inclusion relation ([1], [2]). It is also a problem interesting in itself, as can be seen from [4], where the author has described the ordering $\subset_h$ in the class of all separable and scattered metric spaces. We consider the ordering $\subset_h$ in the class of all scattered P-spaces of weight ω_1, where a P-*space* is a regular space such that its G_δ sets are open. Moreover, in this case it suffices to examine subspaces of the ordinal number ω_2, as every scattered P-space of weight ω_1 is homeomorphic to such a subset.

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[2] A. Dow, *More topological partition relations on* $\beta\omega$, Topology Appl. 259 (2019), 50–66.

[3] M. Fréchet, *Les dimensions d'un ensemble abstrait*, Math. Ann. 68 (1910), no. 2, 145–168.

[4] W. D. Gillam, *Embeddable properties of countable metric spaces*, Topology Appl. 148 (2005), no. 1–3, 63–82.

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[6] W. Sierpiński, *General topology*, Translated by C. Cecilia Krieger. Mathematical Expositions, No. 7, University of Toronto Press, Toronto, 1952.