Partitioning the real line into Borel sets

Will Brian

wbrian@uncc.edu

For which infinite cardinals κ is there a partition of the real line $\mathbb{R}$ into precisely κ Borel sets?

Hausdorff famously proved that there is a partition of $\mathbb{R}$ into $\aleph_1$ Borel sets. The main theorem presented in this talk is that, other than Hausdorff's result, the spectrum of possible sizes of partitions of $\mathbb{R}$ into Borel sets can be fairly arbitrary. For example, given any $A \subseteq \omega$ with $0, 1 \in A$, there is a forcing extension in which $A = \{n :$ there is a partition of $\mathbb{R}$ into $\aleph_n$ Borel sets$\}$.

We also look at the corresponding question for partitions of $\mathbb{R}$ into closed sets. We show that, like with partitions into Borel sets, the set of all uncountable κ such that there is a partition of $\mathbb{R}$ into precisely κ closed sets can be fairly arbitrary.