A proof of the Tree Alternative Conjecture for the Topological Minor Relation

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The Tree Alternative Conjecture states that the equivalence class of any tree (rooted or unrooted) under mutual embeddability is either 1 or infinite. We prove the analogous for the topological minor relation.

Theorem. *For any tree T*

(1) $|T| = 1$ *or* $|T| \geq \aleph_0$, *and*

(2) *for any* $r \in V(T)$, $|(T, r)| = 1$ *or* $|(T, r)| \geq \aleph_0$.

The above is proved by means of stratifying all trees into two complementary categories: those containing all *large* and those containing *small* trees. We then establish the following

Theorem. *For any large tree T,*

(1) $|T| \geq 2^{\aleph_0}$ *and*

(2) *for any* $r \in V(T)$, $|(T, r)| \geq 2^{\aleph_0}$.

Theorem. *For any small tree T,*

(1) $|T| = 1$ *or* $|T| \geq \aleph_0$, *and*

(2) *for any* $r \in V(T)$, $|(T, r)| = 1$ *or* $|(T, r)| \geq \aleph_0$.