On the cardinality of a power homogeneous compactum

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In 2006 de la Vega showed that the cardinality of a homogeneous compactum X is at most $2^{t(X)}$. This was followed in 2007 by Arhangel'skii, van Mill, and Ridderbos who showed the same cardinality bound holds if X is a power homogeneous compactum. In this talk we show that the tightness $t(X)$ can be replaced with $at(X)\pi\chi(X)$, where the almost tightness $at(X)$ satisfies the property $wt(X) \leq at(X) \leq t(X)$. As $\pi\chi(X) \leq t(X)$ for a compactum X and $at(X) \leq t(X)$ for any space, this gives a formal improvement of the result of Arhangel'skii, van Mill, and Ridderbos.

Power homogeneity is used through the following key result. Note a set G is a G^c_κ-set of a space Y if there exists a family of open sets $\mathcal{U}$ in Y such that $|\mathcal{U}| \leq \kappa$ and $G = \bigcap \mathcal{U} = \bigcap_{U \in \mathcal{U}} \overline{U}$.

Let X be a power homogeneous Hausdorff space where $\pi\chi(X) \leq \kappa$. Suppose there exists a nonempty G^c_κ-set G and a set $H \in [X]^{\leq\kappa}$ such that $G \subseteq \overline{H}$. Then there exists a cover $\mathcal{G}$ of X consisting of G^c_κ-sets such that for all $G \in \mathcal{G}$ there exists $H_G \in [X]^{\leq\kappa}$ such that $G \subseteq \overline{H_G}$.

Another component of the proof of the main cardinality bound involves the notion of a T-free sequence, a stronger type of free sequence. We show that a compact subset of a space X contains no T-free sequence of length κ^+ when $\kappa = at(X)$. Further components involve results concerning the weak tightness $wt(X)$ and a cardinality bound for power homogeneous spaces due to Ridderbos.