Entropy of amenable monoid actions

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For a right action $K \overset{\rho}{\curvearrowleft} S$ of a cancellative right amenable monoid S on a compact Hausdorff space K, we build its *Ore colocalization* $K^* \overset{\rho^*}{\curvearrowleft} G$, where K^* is a compact space and G is the group of left fractions of S. This construction preserves the topological entropy (i.e., $h_{\mathrm{top}}(\rho^*) = h_{\mathrm{top}}(\rho)$) and linearity of the action.

Similarly, for a left linear action $S \overset{\lambda}{\curvearrowright} X$ on a discrete Abelian group X, we construct its *Ore localization* $G \overset{\lambda^*}{\curvearrowright} X^*$, which is linear and preserves the algebraic entropy h_{alg} (i.e., $h_{\mathrm{alg}}(\lambda^*) = h_{\mathrm{alg}}(\lambda)$). Moreover, if $K \overset{\rho}{\curvearrowleft} S$ a right linear action with K a compact Abelian group and $S \overset{\rho^\wedge}{\curvearrowright} X$ is the dual left action on the discrete Pontryagin dual $X := K^\wedge$, then the Ore localization of $\rho^\wedge$ is conjugated to dual of the Ore colocalization $K^* \overset{\rho}{\curvearrowleft} G$. Using this fact, we prove the useful equality $h_{\mathrm{top}}(\rho) = h_{\mathrm{alg}}(\rho^\wedge)$, known also as *Bridge Theorem.*

We obtain an *Addition Theorem for* h_{top} (i.e., for a linear action $K \overset{\rho}{\curvearrowleft} S$ on a compact group K, a ρ-invariant closed subgroup H of K and the left cosets space K/H, $h_{\mathrm{top}}(\rho) = h_{\mathrm{top}}(\rho_H) + h_{\mathrm{top}}(\rho_{K/H})$), as well as a similar *Addition Theorem for* h_{alg}.