On some results about cardinal inequalities for topological spaces

Ivan S. Gotchev

gotchevi@ccsu.edu

Two of the most celebrated cardinal inequalities, which are valid for every Hausdorff topological space X, are Arhangel'skiĭ's inequality $|X| \leq 2^{\chi(X)L(X)}$, and Hajnal–Juhász' inequality $|X| \leq 2^{\chi(X)c(X)}$. The two inequalities are important, in particular, because they show that the two pairs of cardinal functions $L(X)$ and $\chi(X)$, and $c(X)$ and $\chi(X)$, respectively, are sufficient to give an upper bound for the cardinality of a Hausdorff topological space. But even Pospíšil's inequality (from 1937) $|X| \leq d(X)^{\chi(X)}$, which is also valid for every Hausdorff space X, gives always the same or a lower upper bound for the cardinality of X than the above two inequalities. This fact explains why there are so many improvements in the literature of these two inequalities.

In this talk we will compare some known results about cardinal inequalities for topological spaces and we will mention some new improvements (of some) of the above inequalities. In particular, we will mention Gotchev–Tkachenko–Tkachuk's inequality from 2016 that $|X| \leq \pi w(X)^{\mathrm{ot(X)}\psi_{\mathrm{c}}(\mathrm{X})}$, where ot(X) is the o-tightness of X. This inequality is valid for every Hausdorff space X and it improves not only Hajnal–Juhász' inequality but also Sun's inequality $|X| \leq \pi\chi(X)^{c(X)\psi_{\mathrm{c}}(X)}$, where $\pi\chi(X)$ is the π-character of X.

We will finish with our recent result that $|X| \leq \pi w(X)^{\mathrm{dot(X)}\cdot\psi_{\mathrm{c}}(\mathrm{X})}$, where $\pi w(X)$ is the π-weight and dot(X) is the dense o-tightness of X, which improves all of the above-mentioned cardinal inequalities.