New applications of Ψ-spaces in analysis

Piotr Koszmider[1]

piotr.koszmider@impan.pl

$\Psi_{\mathcal{A}}$ stands for the standard locally compact noncompact Ψ-space induced by an almost disjoint family $\mathcal{A}$. I will review a few new applications of Ψ-spaces in functional analysis. The first one involves $\mathbb{N} \times \mathbb{N}$ matrices acting on $C_0(\Psi_{\mathcal{A}})$ which generalize partitioners of almost disjoint families or clopen subsets of $\Psi_{\mathcal{A}}$ and provide new examples in algebras of operators ([1]). The second one involves continuous functions from $\Psi_{\mathcal{A}}$ into 2×2 matrices with pointwise noncommutative multiplication and concerns C*-algebras ([2]). The third one is an equivalent renorming of $C_0(\Psi_{\mathcal{A}})$ which becomes the first example of a nonseparable Banach space where the unit sphere does not admit an uncountable subset X such that $\|x - y\| > 1$ for all distinct $x, y \in X$ ([3]). All these applications require special combinatorial properties of the almost disjoint families.

[1] P. KOSZMIDER AND N. J. LAUSTSEN, *A Banach space induced by an almost disjoint family, admitting only few operators and decompositions*, Adv. Math., 381 (2021), paper no. 107613, 39 pages.

[2] O. GUZMÁN, M. HRUŠÁK, AND P. KOSZMIDER, *On $\mathbb{R}$-embeddability of almost disjoint families and Akemann-Doner C*-algebras*, Fund. Math., 254 (2021), pp. 15–47.

[3] P. KOSZMIDER, *Banach spaces in which large subsets of spheres concentrate.* arXiv:2104.05335.

TOPOSYM

Prague 2022