The Menger property is l-invariant

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For a Tychonoff space X, by $C_p(X)$ we denote the space of all continuous real-valued functions on X endowed with the topology of pointwise convergence. Recall that a space X has the *Menger property* if for every sequence $(\mathcal{U}_n)_{n\in\mathbb{N}}$ of open covers of X, there is a sequence $(\mathcal{V}_n)_{n\in\mathbb{N}}$ such that for every n, $\mathcal{V}_n$ is a finite subfamily of $\mathcal{U}_n$ and the family $\bigcup_{n\in\mathbb{N}} \mathcal{V}_n$ covers X.

An old question of A.V. Arhangel'skii asks if the Menger property of X is preserved by homeomorphisms of the space $C_p(X)$. A similar question can also be asked for linear homeomorphisms of $C_p(X)$-spaces. In 2020 M. Sakai gave the affirmative answer in the linear case under an additional assumption on X. We show that the answer in the linear case is affirmative in the full generality. Our method can also be applied to prove analogous theorems for other, related covering type properties.