Generic Polish metric spaces

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Given a class $\mathcal{M}$ of finite metric spaces, one can consider a natural infinite game in which two players alternately build a chain $M_0 \subseteq M_1 \subseteq M_2 \subseteq \cdots$ in $\mathcal{M}$. After infinitely many steps one can look at the completion M_∞ of the union of this chain. We say that a Polish space V is $\mathcal{M}$-*generic* if the second player has a strategy making M_∞ isometric to V.

The existence of a generic Polish space over a given class is strictly related to an approximate variant of the weak amalgamation property. This is a somewhat technical weakening of the amalgamation property, discovered several decades ago in model theory, that became relevant in the study of generic automorphisms.

Typical examples of generic metric spaces are the (ultra-)homogeneous ones, namely, those in which every finite isometry extends to a bijective isometry. The most complicated one is the Urysohn space. A metric space is *approximately homogenous* if every isometry between its finite subsets can be approximated by a bijective isometry. It turns out that approximate homogeneity is sufficient for being generic. We shall discuss some other natural variants of homogeneity and injectivity, leading to generic spaces.

The talk is based on a joint work (in progress) with Christian Bargetz, Adam Bartoš, and Franz Luggin.