Δ-spaces X and distinguished spaces $C_p(X)$

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Definition. (G. M. Reed, E. van Douwen) A subset of reals $X \subset \mathbb{R}$ is said to be a Δ-*set* if for every decreasing sequence $\{D_n : n \in \omega\}$ of subsets of X with empty intersection, there is a decreasing sequence $\{V_n : n \in \omega\}$ consisting of open subsets of X, also with empty intersection, and such that $D_n \subset V_n$ for every $n \in \omega$.

Definition. (A. Grothendieck) A locally convex space (lcs) E is called *distinguished* if the strong dual of E (i.e. the topological dual of E endowed with the strong topology) is barrelled.

Theorem. *Let X be a Tychonoff space. A lcs $C_p(X)$ is distinguished if and only if for each $f \in \mathbb{R}^X$ there is a bounded $B \subset C_p(X)$ such that f belongs to the closure of B in $\mathbb{R}^X$.*

We say that a Tychonoff space X is a Δ-*space* if X satisfies property Δ, as in the first Definition above.

Theorem. *Let X be a Tychonoff space. A lcs $C_p(X)$ is distinguished if and only if X is a Δ-space.*

My talk will be devoted to the main results about Δ-spaces which are published recently in several joint works.