The topological end space problem

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End spaces of infinite graphs sit at the interface between topology, graph theory, and group theory. They arise as the boundary of an infinite graph in a standard sense generalising the theory of the Freudenthal boundary developed by Freudenthal and Hopf in the 1940's for infinite groups.

A long-standing quest in infinite graph theory with a rich body of literature seeks to characterise the possible end structures of graphs, with the eventual goal of finding a purely topological characterisation of end spaces:

Question (The topological end space problem)**.** Find a topological characterisation of those spaces that arise as end space of some infinite graph.

In this talk, I will first explain our recent representation theorem for end spaces (see ArXiv:2111.12670 for details):

Theorem (Kurkofka & Pitz)**.** *Every end space is homeomorphic to the end space of a (certain canonical graph on a) special order tree.*

I will then describe possible directions of how our representation theorem may be used in attacking the topological end space problem.