Weak* derived sets

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The weak* derived set $A^{(1)}$ of a subset A of a dual Banach space X^* is the set of weak* limits of bounded nets in A. It is well known that a convex subset of a dual Banach space is weak* closed if and only if it equals its weak* derived set. In genaral, taking weak* derived set is not an idempotent operation – it can happen that $A^{(1)}$ is a proper subset of $\left(A^{(1)}\right)^{(1)}$. This inspires the definition of iterated weak* derived sets. The order of A is then defined to be the least ordinal for which the iteration stabilizes. M. Ostrovskii provided the complete description of possible orders of subspaces of duals of separable non-quasi-reflexive spaces. In this talk we will present some partial results concerning orders of convex subsets of duals of non-reflexive spaces. We also present another special result motivated by the study of extension problems for holomorphic functions on dual Banach spaces. We show that for any non-quasi-reflexive Banach space X containing an infinite-dimensional subspace with separable dual and for any countable non-limit ordinal α we can always find a subspace A of X^* such that $A^{(\alpha)}$ is a proper norm dense subspace of X^*.