On stability and weight of Lindelöf P-groups

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If every G_δ-set in a space X is open, we say that it is a *P-space*. Similarly, a topological group is a *P-group* if it is a P-space. The class of Lindelöf P-groups, which is similar to the class of compact topological groups in terms of permanence properties, is the focus of our presentation.

First, we consider the question of whether every Lindelöf P-group is τ-stable, for a given infinite cardinal τ. It is well known that the answer is affirmative for $\tau = \aleph_0$ and $\tau = \aleph_1$. We extend this conclusion to a proper class of cardinals τ, including those satisfying the equality $\tau^\omega = \tau$. We deduce, for example, that every Lindelöf P-group is $\aleph_n$-stable, for each $n \in \omega$.

Second, we look at the problem of estimating a gap between the weight and i-weight of a Lindelöf P-group G. If the cardinal $\tau = iw(G)$ is either $\aleph_n$ for some $n \in \omega$ or fulfills $\tau^\omega = \tau$, it is not difficult to demonstrate that the two cardinal functions coincide. In general, however, a gap can be quite big. According to our best knowledge, if $iw(G) = \aleph_\omega$, then $w(G) < \aleph_{\omega_4}$ and $w(G) \leq (\tau_\omega)^\omega$.

Our entire analysis is based on the family $[\tau]^\omega$ of countable subsets of an uncountable cardinal τ partially ordered by inclusion.

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