On sequentiality of Polish topologies

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Gutierres showed in ZF that if the usual topology on the real line is sequential, then every infinite set of reals has a countable infinite subset. Using the method of balanced forcing, I show that the opposite implication is not provable, answering his question.

Theorem. *Relative to an inaccessible cardinal, it is consistent with ZF that every infinite set has a countable infinite subset, yet the topology of the real line is not sequential.*

The ease of the proof suggests that one may be able to (consistently with ZF) separate sequentiality of topologies on various Polish spaces. For example:

Question. Is it consistent with ZF that the topology of Euclidean line is sequential, while that of the Euclidean plane is not?

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