Embedding of the Higson compactification into the product of adelic solenoids

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Prague, July 25, 2022

Higson compactification

- DEFINITION. If (X, d) is a metric space, and $f : X \to \mathbb{R}$ is a bounded continuous function, we say that f is *slowly oscillating* if
$$\lim_{x\to\infty} diam(f(B_r(x))) = 0$$
for any fixed r and x tending to infinity in X, i.e. $d(x, x_0) \to \infty$ for some $x_0 \in X$.

- The *Higson compactification* $\overline{X}$ of X is the smallest one containing X densely so that all bounded slowly oscillating functions extend to $\overline{X}$.

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Higson compactification

- Let C_h be the set of all bounded slowly oscillating functions. Then the Higson compactification of X can be obtained by taking the closure of the image of the embedding

$$(f)_{f \in C_h} : X \to \prod_{f \in C_h} [inf(f), sup(f)] \cong I^{C_h}.$$

- The Higson compactification is similar to the Stone-Čech.
- On the other hand, $\bar{\mathbb{Z}} \setminus \mathbb{Z} = \bar{\mathbb{R}} \setminus \mathbb{R}$ but $\beta\mathbb{Z} \setminus \mathbb{Z} \neq \beta\mathbb{R} \setminus \mathbb{R}$.

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$$\check{H}^i(\overline{X}) = 0 \quad \text{for} \quad i > 0.$$

- A manifold is called *aspherical* if its universal covering is contractible.
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Relation to Novikov conjecture

- Assembly map: $\alpha : K_*(B\pi) \to K_*(\mathbb{C}_r\pi)$
- Analytic Novikov conjecture: α is a monomorphism.
- Baum-Connes conjecture: α is an isomorphism.
- Coarse assembly map: $A : K_*^{lf}(E\pi) \to K_*(C^*_{Roe}(E\pi))$.
- Coarse Baum-Connes conjecture: A is an isomorphism.
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Coarse assembly map

- There is a commutative triangle

$$\begin{array}{ccc} K_*(\overline{X}, \nu X) & \xrightarrow{\quad A \quad} & K_*(C^*_{Roe}(X)) \;. \\ & {\scriptstyle \partial}\searrow \qquad \swarrow & \\ & K_{*-1}(\nu X) & \end{array}$$

  where νX is the Higson corona: $\overline{X} = X \cup \nu X$.
- Thus, if ∂ is injective, then A is injective.

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- The exact sequence of the pair $(\overline{X}, \nu X)$ implies that if $\bar{X}$ is acyclic, then ∂ is an isomorphism.
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Higson conjecture for 1-cohomology

- Theorem. (Keesling) $\check{H}^1(\overline{X}) \neq 0$ *for any connected unbounded metric space* X.
- Recall that $\check{H}^1(X) = [X, S^1]$.
- For $X = \mathbb{R}$ a nontrivial class is given by $f : \mathbb{R} \to S^1$ with decaying variation, $f(t) = exp(2\pi i\sqrt{t})$. Then $\bar{f} : \overline{\mathbb{R}} \to S^1$ cannot have a lift ($\sqrt{t}$ is unbounded).

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Main Result

Theorem 1.(Dr.-Keesling)

(Dr.-Keesling) *Every simply connected proper geodesic metric space X admits an embedding of its Higson compactification into the product of adelic solenoids*

$$F : \bar{X} \to \prod_{\mathcal{A}} \Sigma_\infty$$

that induces an isomorphism of 1-dimensional Čech cohomology.

Solenoids

- The *p-adic solenoid* Σ_p is the inverse limit:

$$\Sigma_p = \lim_{\leftarrow}\{S^1 \overset{p}{\leftarrow} S^1 \overset{p}{\leftarrow} S^1 \leftarrow \cdots\}$$

  of unit circles $S^1 \subset \mathbb{C}$ where the bonding maps are $z \mapsto z^p$.
- The universal cover $\mathbb{R} \to S^1$ lifts to an injective group homomorphism $\mathbb{R} \to \Sigma_p$.
- The kernel of $\Sigma_p \to S^1$ is the group of *p-adic integers* $\mathbb{A}_p$.
- Note that $\mathbb{A}_p \cap \mathbb{R} = \mathbb{Z}$.
- Clearly, $\Sigma_p = (\mathbb{R} \times \mathbb{A}_p)/\mathbb{Z}$ for the diagonal embedding $\mathbb{Z} \to \mathbb{R} \times \mathbb{A}_p$.

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- The adelic solenoid can be defined as

$$\Sigma_\infty = (\mathbb{R} \times \hat{\mathbb{Z}})/\mathbb{Z}$$

  where $\hat{\mathbb{Z}}$ is the profinite completion of the integers and $\mathbb{Z} \to \mathbb{R} \times \hat{\mathbb{Z}}$ is the diagonal map.
- All the properties of p-adic solenoids hold for Σ_∞.
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Corollary.

For simply connected proper geodesic metric space X,

$$\check{H}^1(\overline{X};\mathbb{Z}_p)=0$$

for all p.

Higson conjecture mod p

- Mod p Higson Conjecture. *$\check{H}^i(\overline{X};\mathbb{Z}_p) = 0$ for $i > 0$ for the universal coverings X of aspherical manifolds.*
- The above Corollary states that the mod 2 Higson conjecture holds true for $i = 1$.
- By a theorem of Calder and Siegel, the mod p Higson conjecture holds for the Stone-Čech compactification.

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Theorem (Dr.-Ferry-Weinberger)

The mod 2 Higson conjecture for $X = E\Gamma$ implies the Novikov conjecture for the group Γ.

- Here $E\Gamma$ is the universal covering of the classifying space $B\Gamma = K(\Gamma, 1)$.
- Note that for $\Gamma = \pi_1(M)$ in case of an aspherical n-manifold M, the space $E\Gamma \times \mathbb{R}$ is homeomorphic to $\mathbb{R}^{n+1}$.

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Satellite Results

Theorem 2.

For each discrete group Γ with finite complex $B\Gamma$ there is a Γ-equivariant embedding of the Higson compactification of $E\Gamma$ into the product of adelic solenoids

$$F : \overline{E\Gamma} \to \prod \Sigma_{\infty}$$

that induces an epimorphism of the integral 1-dimensional Čech cohomology.

Satellite Results

Theorem 3.

For any p every simply connected proper geodesic metric space X admits an embedding of its Higson compactification into the product of p-adic solenoids

$$F : \bar{X} \to \prod_{\mathcal{A}} \Sigma_p$$

that induces a rational isomorphism of 1-cohomology.

Essential embedding into the product

- We call a map $f : X \to Z$ *essential* if every map $g : X \to Z$ homotopic to f is surjective.
- We call a subset $X \subset \prod Z_\alpha$ *essential* if its projection on each factor $p_\alpha : X \to Z_\alpha$ is essential.
- Corollary of Main Resut: *For any p every simply connected proper geodesic metric space X admits an essential embedding of its Higson compactification into a product of p-adic solenoids.*

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Knaster Continua

- We define a continuum $K_p = \Sigma_p / \sim$ to be the quotient space under the identification $x \sim -x$.
- A theorem of Bellamy says that K_2 is homeomorphic to the Knaster continuum, also known as the Bucket handle continuum.
- Proposition. *Any surjective map $f : Y \to K_p$ of a connected compact Hausdorff space is essential.*
- Question. Is it true that for any indecomposable continuum X every surjective map $f : Y \to X$ of a compact connected space is essential?

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Embedding into the product of Knaster continua

Theorem 4.

For any p and any simply connected finite dimensional proper geodesic metric space X its Higson compactification can be essentially embedded into the product of continua K_p.

THANK YOU!!!