The double density spectrum of a topological space

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It is consistent to have a separable compactum C such that $\{\aleph_\alpha : 2 \leq \alpha < \omega_1\} \subset dd(C)$ but $\aleph_1, \aleph_{\omega_1} \notin dd(C)$, hence $dd(C)$ is not ω_1-closed.

PROBLEMS. (i) Let S be an ω-closed set of infinite cardinals such that $\sup S = \max S \leq 2^{\min S}$. Is there a ompactum K s.t. $dd(K) = S$?
(ii) Does $2^\kappa > \kappa^+$ imply the existence of a ompactum K of density κ s.t. $dd(K) \neq [\kappa, \pi(K)]$? How about $\kappa = \omega$?

THANK YOU FOR YOUR ATTENTION!