New applications of Ψ-spaces in analysis

Piotr Koszmider

Institute of Mathematics of the Polish Academy of Sciences

Almost disjoint families and Ψ-spaces

Almost disjoint families and Ψ-spaces

- A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is *almost disjoint* if

Almost disjoint families and Ψ-spaces

- A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is *almost disjoint* if

Almost disjoint families and Ψ-spaces

- A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is *almost disjoint* if any two distinct elements of $\mathcal{A}$ have finite intersection.

Almost disjoint families and Ψ-spaces

- A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is *almost disjoint* if any two distinct elements of $\mathcal{A}$ have finite intersection.
- $$\Psi_{\mathcal{A}} = \mathbb{N} \cup \{x_A : A \in \mathcal{A}\},$$

Almost disjoint families and Ψ-spaces

- A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is *almost disjoint* if any two distinct elements of $\mathcal{A}$ have finite intersection.
-
$$\Psi_{\mathcal{A}} = \mathbb{N} \cup \{x_A : A \in \mathcal{A}\},$$

Almost disjoint families and Ψ-spaces

- A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is *almost disjoint* if any two distinct elements of $\mathcal{A}$ have finite intersection.
- $$\Psi_{\mathcal{A}} = \mathbb{N} \cup \{x_A : A \in \mathcal{A}\},$$
  - elements of $\mathbb{N}$ are isolated,

Almost disjoint families and Ψ-spaces

- A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is *almost disjoint* if any two distinct elements of $\mathcal{A}$ have finite intersection.
- $$\Psi_{\mathcal{A}} = \mathbb{N} \cup \{x_A : A \in \mathcal{A}\},$$
  - elements of $\mathbb{N}$ are isolated,

Almost disjoint families and Ψ-spaces

- A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is *almost disjoint* if any two distinct elements of $\mathcal{A}$ have finite intersection.
- $$\Psi_{\mathcal{A}} = \mathbb{N} \cup \{x_A : A \in \mathcal{A}\},$$
  - elements of $\mathbb{N}$ are isolated,
  - neighbourhoods of x_As are

Almost disjoint families and Ψ-spaces

- A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is *almost disjoint* if any two distinct elements of $\mathcal{A}$ have finite intersection.
- $$\Psi_{\mathcal{A}} = \mathbb{N} \cup \{x_A : A \in \mathcal{A}\},$$
    - elements of $\mathbb{N}$ are isolated,
    - neighbourhoods of x_As are

Almost disjoint families and Ψ-spaces

- A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is *almost disjoint* if any two distinct elements of $\mathcal{A}$ have finite intersection.
-
$$\Psi_{\mathcal{A}} = \mathbb{N} \cup \{x_A : A \in \mathcal{A}\},$$
  - elements of $\mathbb{N}$ are isolated,
  - neighbourhoods of x_As are
$$V_A(n) = (A \setminus n) \cup \{x_A\}$$

Almost disjoint families and Ψ-spaces

- A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is *almost disjoint* if any two distinct elements of $\mathcal{A}$ have finite intersection.
- $$\Psi_{\mathcal{A}} = \mathbb{N} \cup \{x_A : A \in \mathcal{A}\},$$
  - elements of $\mathbb{N}$ are isolated,
  - neighbourhoods of x_As are
  $$V_A(n) = (A \setminus n) \cup \{x_A\}$$
  for $n \in \mathbb{N}$,

Almost disjoint families and Ψ-spaces

- A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is *almost disjoint* if any two distinct elements of $\mathcal{A}$ have finite intersection.
- $$\Psi_{\mathcal{A}} = \mathbb{N} \cup \{x_A : A \in \mathcal{A}\},$$
  - elements of $\mathbb{N}$ are isolated,
  - neighbourhoods of x_As are
  $$V_A(n) = (A \setminus n) \cup \{x_A\}$$
  for $n \in \mathbb{N}$, so $\{k : k \in A\} \to x_A$.

Almost disjoint families and Ψ-spaces

- A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is *almost disjoint* if any two distinct elements of $\mathcal{A}$ have finite intersection.
- $$\Psi_{\mathcal{A}} = \mathbb{N} \cup \{x_A : A \in \mathcal{A}\},$$
  - elements of $\mathbb{N}$ are isolated,
  - neighbourhoods of x_As are
  $$V_A(n) = (A \setminus n) \cup \{x_A\}$$
  for $n \in \mathbb{N}$, so $\{k : k \in A\} \to x_A$.
- $$C_0(\Psi_{\mathcal{A}}) = \{f \in C(\Psi_{\mathcal{A}}, \mathbb{R}) : \forall \varepsilon > 0\ f^{-1}[[-\varepsilon, \varepsilon]] \text{ is compact}\}$$

Almost disjoint families and Ψ-spaces

- A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is *almost disjoint* if any two distinct elements of $\mathcal{A}$ have finite intersection.
- $$\Psi_{\mathcal{A}} = \mathbb{N} \cup \{x_A : A \in \mathcal{A}\},$$
  - elements of $\mathbb{N}$ are isolated,
  - neighbourhoods of x_As are
  $$V_A(n) = (A \setminus n) \cup \{x_A\}$$
  for $n \in \mathbb{N}$, so $\{k : k \in A\} \to x_A$.
- $$C_0(\Psi_{\mathcal{A}}) = \{f \in C(\Psi_{\mathcal{A}}, \mathbb{R}) : \forall \varepsilon > 0\ f^{-1}[[-\varepsilon, \varepsilon]] \text{ is compact}\}$$

Almost disjoint families and Ψ-spaces

- A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is *almost disjoint* if any two distinct elements of $\mathcal{A}$ have finite intersection.
- $$\Psi_{\mathcal{A}} = \mathbb{N} \cup \{x_A : A \in \mathcal{A}\},$$
  - elements of $\mathbb{N}$ are isolated,
  - neighbourhoods of x_As are
  $$V_A(n) = (A \setminus n) \cup \{x_A\}$$
  for $n \in \mathbb{N}$, so $\{k : k \in A\} \to x_A$.
- $$C_0(\Psi_{\mathcal{A}}) = \{f \in C(\Psi_{\mathcal{A}}, \mathbb{R}) : \forall \varepsilon > 0\ f^{-1}[[-\varepsilon, \varepsilon]] \text{ is compact}\}$$
- $A \subseteq^* B$ means that $A \setminus B$ is finite.

Almost disjoint families and Ψ-spaces

- A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is *almost disjoint* if any two distinct elements of $\mathcal{A}$ have finite intersection.
- $$\Psi_{\mathcal{A}} = \mathbb{N} \cup \{x_A : A \in \mathcal{A}\},$$
  - elements of $\mathbb{N}$ are isolated,
  - neighbourhoods of x_As are
  $$V_A(n) = (A \setminus n) \cup \{x_A\}$$
  for $n \in \mathbb{N}$, so $\{k : k \in A\} \to x_A$.
- $$C_0(\Psi_{\mathcal{A}}) = \{f \in C(\Psi_{\mathcal{A}}, \mathbb{R}) : \forall \varepsilon > 0\ f^{-1}[[-\varepsilon, \varepsilon]] \text{ is compact}\}$$
- $A \subseteq^* B$ means that $A \setminus B$ is finite.

Almost disjoint families and Ψ-spaces

- A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is *almost disjoint* if any two distinct elements of $\mathcal{A}$ have finite intersection.
- $$\Psi_{\mathcal{A}} = \mathbb{N} \cup \{x_A : A \in \mathcal{A}\},$$
  - elements of $\mathbb{N}$ are isolated,
  - neighbourhoods of x_As are
  $$V_A(n) = (A \setminus n) \cup \{x_A\}$$
  for $n \in \mathbb{N}$, so $\{k : k \in A\} \to x_A$.
- $$C_0(\Psi_{\mathcal{A}}) = \{f \in C(\Psi_{\mathcal{A}}, \mathbb{R}) : \forall \varepsilon > 0\ f^{-1}[[-\varepsilon, \varepsilon]] \text{ is compact}\}$$
- $A \subseteq^* B$ means that $A \setminus B$ is finite.

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.
- $A \subseteq \mathbb{N}$ is called a *partitioner* of $\mathcal{A}$ if

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.
- $A \subseteq \mathbb{N}$ is called a *partitioner* of $\mathcal{A}$ if

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.
- $A \subseteq \mathbb{N}$ is called a *partitioner* of $\mathcal{A}$ if it separates some subfamily $\mathcal{B}$ of $\mathcal{A}$ from its complement $\mathcal{A} \setminus \mathcal{B}$.

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.
- $A \subseteq \mathbb{N}$ is called a *partitioner* of $\mathcal{A}$ if it separates some subfamily $\mathcal{B}$ of $\mathcal{A}$ from its complement $\mathcal{A} \setminus \mathcal{B}$.
- $\mathcal{A}$ is called $\mathbb{R}$-*embeddable*

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.
- $A \subseteq \mathbb{N}$ is called a *partitioner* of $\mathcal{A}$ if it separates some subfamily $\mathcal{B}$ of $\mathcal{A}$ from its complement $\mathcal{A} \setminus \mathcal{B}$.
- $\mathcal{A}$ is called $\mathbb{R}$*-embeddable*
  - iff there is a function $f : \mathbb{N} \to \mathbb{Q}$ such that for each $A \in \mathcal{A}$ the set $f[A]$ is the set of all terms of a sequence convergent to $r_A \in \mathbb{R} \setminus \mathbb{Q}$

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.
- $A \subseteq \mathbb{N}$ is called a *partitioner* of $\mathcal{A}$ if it separates some subfamily $\mathcal{B}$ of $\mathcal{A}$ from its complement $\mathcal{A} \setminus \mathcal{B}$.
- $\mathcal{A}$ is called $\mathbb{R}$*-embeddable*
  - iff there is a function $f : \mathbb{N} \to \mathbb{Q}$ such that for each $A \in \mathcal{A}$ the set $f[A]$ is the set of all terms of a sequence convergent to $r_A \in \mathbb{R} \setminus \mathbb{Q}$

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.
- $A \subseteq \mathbb{N}$ is called a *partitioner* of $\mathcal{A}$ if it separates some subfamily $\mathcal{B}$ of $\mathcal{A}$ from its complement $\mathcal{A} \setminus \mathcal{B}$.
- $\mathcal{A}$ is called *$\mathbb{R}$-embeddable*
  - iff there is a function $f : \mathbb{N} \to \mathbb{Q}$ such that for each $A \in \mathcal{A}$ the set $f[A]$ is the set of all terms of a sequence convergent to $r_A \in \mathbb{R} \setminus \mathbb{Q}$ and $r_A \neq r_B$ if $A, B \in \mathcal{A}$ are distinct.

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.
- $A \subseteq \mathbb{N}$ is called a *partitioner* of $\mathcal{A}$ if it separates some subfamily $\mathcal{B}$ of $\mathcal{A}$ from its complement $\mathcal{A} \setminus \mathcal{B}$.
- $\mathcal{A}$ is called $\mathbb{R}$*-embeddable*
  - iff there is a function $f : \mathbb{N} \to \mathbb{Q}$ such that for each $A \in \mathcal{A}$ the set $f[A]$ is the set of all terms of a sequence convergent to $r_A \in \mathbb{R} \setminus \mathbb{Q}$ and $r_A \neq r_B$ if $A, B \in \mathcal{A}$ are distinct.
  - iff there is a continuous $F : \Psi_{\mathcal{A}} \to \mathbb{R}$ such that $F|\{x_A : A \in \mathcal{A}\}$ is injective.

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.
- $A \subseteq \mathbb{N}$ is called a *partitioner* of $\mathcal{A}$ if it separates some subfamily $\mathcal{B}$ of $\mathcal{A}$ from its complement $\mathcal{A} \setminus \mathcal{B}$.
- $\mathcal{A}$ is called *$\mathbb{R}$-embeddable*
  - iff there is a function $f : \mathbb{N} \to \mathbb{Q}$ such that for each $A \in \mathcal{A}$ the set $f[A]$ is the set of all terms of a sequence convergent to $r_A \in \mathbb{R} \setminus \mathbb{Q}$ and $r_A \neq r_B$ if $A, B \in \mathcal{A}$ are distinct.
  - iff there is a continuous $F : \Psi_{\mathcal{A}} \to \mathbb{R}$ such that $F|\{x_A : A \in \mathcal{A}\}$ is injective.
- $\mathcal{A}$ is called *Q-family*

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.
- $A \subseteq \mathbb{N}$ is called a *partitioner* of $\mathcal{A}$ if it separates some subfamily $\mathcal{B}$ of $\mathcal{A}$ from its complement $\mathcal{A} \setminus \mathcal{B}$.
- $\mathcal{A}$ is called *$\mathbb{R}$-embeddable*
  - iff there is a function $f : \mathbb{N} \to \mathbb{Q}$ such that for each $A \in \mathcal{A}$ the set $f[A]$ is the set of all terms of a sequence convergent to $r_A \in \mathbb{R} \setminus \mathbb{Q}$ and $r_A \neq r_B$ if $A, B \in \mathcal{A}$ are distinct.
  - iff there is a continuous $F : \Psi_{\mathcal{A}} \to \mathbb{R}$ such that $F|\{x_A : A \in \mathcal{A}\}$ is injective.
- $\mathcal{A}$ is called *Q-family*
  - iff every subfamily $\mathcal{B} \subseteq \mathcal{A}$ can be separated from its complement $\mathcal{A} \setminus \mathcal{B}$.

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.
- $A \subseteq \mathbb{N}$ is called a *partitioner* of $\mathcal{A}$ if it separates some subfamily $\mathcal{B}$ of $\mathcal{A}$ from its complement $\mathcal{A} \setminus \mathcal{B}$.
- $\mathcal{A}$ is called *$\mathbb{R}$-embeddable*
  - iff there is a function $f : \mathbb{N} \to \mathbb{Q}$ such that for each $A \in \mathcal{A}$ the set $f[A]$ is the set of all terms of a sequence convergent to $r_A \in \mathbb{R} \setminus \mathbb{Q}$ and $r_A \neq r_B$ if $A, B \in \mathcal{A}$ are distinct.
  - iff there is a continuous $F : \Psi_{\mathcal{A}} \to \mathbb{R}$ such that $F|\{x_A : A \in \mathcal{A}\}$ is injective.
- $\mathcal{A}$ is called *Q-family*
  - iff every subfamily $\mathcal{B} \subseteq \mathcal{A}$ can be separated from its complement $\mathcal{A} \setminus \mathcal{B}$.

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.
- $A \subseteq \mathbb{N}$ is called a *partitioner* of $\mathcal{A}$ if it separates some subfamily $\mathcal{B}$ of $\mathcal{A}$ from its complement $\mathcal{A} \setminus \mathcal{B}$.
- $\mathcal{A}$ is called $\mathbb{R}$*-embeddable*
  - iff there is a function $f : \mathbb{N} \to \mathbb{Q}$ such that for each $A \in \mathcal{A}$ the set $f[A]$ is the set of all terms of a sequence convergent to $r_A \in \mathbb{R} \setminus \mathbb{Q}$ and $r_A \neq r_B$ if $A, B \in \mathcal{A}$ are distinct.
  - iff there is a continuous $F : \Psi_{\mathcal{A}} \to \mathbb{R}$ such that $F|\{x_A : A \in \mathcal{A}\}$ is injective.
- $\mathcal{A}$ is called *Q-family*
  - iff every subfamily $\mathcal{B} \subseteq \mathcal{A}$ can be separated from its complement $\mathcal{A} \setminus \mathcal{B}$.
  - iff $\Psi_{\mathcal{A}}$ is normal.

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.
- $A \subseteq \mathbb{N}$ is called a *partitioner* of $\mathcal{A}$ if it separates some subfamily $\mathcal{B}$ of $\mathcal{A}$ from its complement $\mathcal{A} \setminus \mathcal{B}$.
- $\mathcal{A}$ is called *$\mathbb{R}$-embeddable*
  - iff there is a function $f : \mathbb{N} \to \mathbb{Q}$ such that for each $A \in \mathcal{A}$ the set $f[A]$ is the set of all terms of a sequence convergent to $r_A \in \mathbb{R} \setminus \mathbb{Q}$ and $r_A \neq r_B$ if $A, B \in \mathcal{A}$ are distinct.
  - iff there is a continuous $F : \Psi_{\mathcal{A}} \to \mathbb{R}$ such that $F|\{x_A : A \in \mathcal{A}\}$ is injective.
- $\mathcal{A}$ is called *Q-family*
  - iff every subfamily $\mathcal{B} \subseteq \mathcal{A}$ can be separated from its complement $\mathcal{A} \setminus \mathcal{B}$.
  - iff $\Psi_{\mathcal{A}}$ is normal.

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.
- $A \subseteq \mathbb{N}$ is called a *partitioner* of $\mathcal{A}$ if it separates some subfamily $\mathcal{B}$ of $\mathcal{A}$ from its complement $\mathcal{A} \setminus \mathcal{B}$.
- $\mathcal{A}$ is called *$\mathbb{R}$-embeddable*
  - iff there is a function $f : \mathbb{N} \to \mathbb{Q}$ such that for each $A \in \mathcal{A}$ the set $f[A]$ is the set of all terms of a sequence convergent to $r_A \in \mathbb{R} \setminus \mathbb{Q}$ and $r_A \neq r_B$ if $A, B \in \mathcal{A}$ are distinct.
  - iff there is a continuous $F : \Psi_{\mathcal{A}} \to \mathbb{R}$ such that $F|\{x_A : A \in \mathcal{A}\}$ is injective.
- $\mathcal{A}$ is called *Q-family*
  - iff every subfamily $\mathcal{B} \subseteq \mathcal{A}$ can be separated from its complement $\mathcal{A} \setminus \mathcal{B}$.
  - iff $\Psi_{\mathcal{A}}$ is normal.
- $\mathcal{A}$ is *Mrówka's family* if $\Psi^* = \beta\Psi_{\mathcal{A}} \setminus \Psi_{\mathcal{A}}$ consist just of one point.

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.
- $A \subseteq \mathbb{N}$ is called a *partitioner* of $\mathcal{A}$ if it separates some subfamily $\mathcal{B}$ of $\mathcal{A}$ from its complement $\mathcal{A} \setminus \mathcal{B}$.
- $\mathcal{A}$ is called *$\mathbb{R}$-embeddable*
  - iff there is a function $f : \mathbb{N} \to \mathbb{Q}$ such that for each $A \in \mathcal{A}$ the set $f[A]$ is the set of all terms of a sequence convergent to $r_A \in \mathbb{R} \setminus \mathbb{Q}$ and $r_A \neq r_B$ if $A, B \in \mathcal{A}$ are distinct.
  - iff there is a continuous $F : \Psi_{\mathcal{A}} \to \mathbb{R}$ such that $F|\{x_A : A \in \mathcal{A}\}$ is injective.
- $\mathcal{A}$ is called *Q-family*
  - iff every subfamily $\mathcal{B} \subseteq \mathcal{A}$ can be separated from its complement $\mathcal{A} \setminus \mathcal{B}$.
  - iff $\Psi_{\mathcal{A}}$ is normal.
- $\mathcal{A}$ is *Mrówka's family* if $\Psi^* = \beta\Psi_{\mathcal{A}} \setminus \Psi_{\mathcal{A}}$ consist just of one point.

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.
- $A \subseteq \mathbb{N}$ is called a *partitioner* of $\mathcal{A}$ if it separates some subfamily $\mathcal{B}$ of $\mathcal{A}$ from its complement $\mathcal{A} \setminus \mathcal{B}$.
- $\mathcal{A}$ is called *$\mathbb{R}$-embeddable*
  - iff there is a function $f : \mathbb{N} \to \mathbb{Q}$ such that for each $A \in \mathcal{A}$ the set $f[A]$ is the set of all terms of a sequence convergent to $r_A \in \mathbb{R} \setminus \mathbb{Q}$ and $r_A \neq r_B$ if $A, B \in \mathcal{A}$ are distinct.
  - iff there is a continuous $F : \Psi_{\mathcal{A}} \to \mathbb{R}$ such that $F|\{x_A : A \in \mathcal{A}\}$ is injective.
- $\mathcal{A}$ is called *Q-family*
  - iff every subfamily $\mathcal{B} \subseteq \mathcal{A}$ can be separated from its complement $\mathcal{A} \setminus \mathcal{B}$.
  - iff $\Psi_{\mathcal{A}}$ is normal.
- $\mathcal{A}$ is *Mrówka's family* if $\Psi^* = \beta\Psi_{\mathcal{A}} \setminus \Psi_{\mathcal{A}}$ consist just of one point.
- $\mathcal{A}$ is *Lusin's family* if no two uncountable subfamilies of $\mathcal{A}$ can be separated.

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.
- $A \subseteq \mathbb{N}$ is called a *partitioner* of $\mathcal{A}$ if it separates some subfamily $\mathcal{B}$ of $\mathcal{A}$ from its complement $\mathcal{A} \setminus \mathcal{B}$.
- $\mathcal{A}$ is called *$\mathbb{R}$-embeddable*
  - iff there is a function $f : \mathbb{N} \to \mathbb{Q}$ such that for each $A \in \mathcal{A}$ the set $f[A]$ is the set of all terms of a sequence convergent to $r_A \in \mathbb{R} \setminus \mathbb{Q}$ and $r_A \neq r_B$ if $A, B \in \mathcal{A}$ are distinct.
  - iff there is a continuous $F : \Psi_{\mathcal{A}} \to \mathbb{R}$ such that $F|\{x_A : A \in \mathcal{A}\}$ is injective.
- $\mathcal{A}$ is called *Q-family*
  - iff every subfamily $\mathcal{B} \subseteq \mathcal{A}$ can be separated from its complement $\mathcal{A} \setminus \mathcal{B}$.
  - iff $\Psi_{\mathcal{A}}$ is normal.
- $\mathcal{A}$ is *Mrówka's family* if $\Psi^* = \beta\Psi_{\mathcal{A}} \setminus \Psi_{\mathcal{A}}$ consist just of one point.
- $\mathcal{A}$ is *Lusin's family* if no two uncountable subfamilies of $\mathcal{A}$ can be separated.

Some properties of almost disjoint families $\mathcal{A}$ and corresponding $\Psi_{\mathcal{A}}$-spaces

- Two subfamilies $\mathcal{B}, \mathcal{C} \subseteq \mathcal{A}$ are *separated* by $A \subseteq \mathbb{N}$ if :
  - If $B \in \mathcal{B}$ then $B \subseteq^* A$.
  - If $C \in \mathcal{C}$ then $C \subseteq^* \mathbb{N} \setminus A$.
- $A \subseteq \mathbb{N}$ is called a *partitioner* of $\mathcal{A}$ if it separates some subfamily $\mathcal{B}$ of $\mathcal{A}$ from its complement $\mathcal{A} \setminus \mathcal{B}$.
- $\mathcal{A}$ is called *$\mathbb{R}$-embeddable*
  - iff there is a function $f : \mathbb{N} \to \mathbb{Q}$ such that for each $A \in \mathcal{A}$ the set $f[A]$ is the set of all terms of a sequence convergent to $r_A \in \mathbb{R} \setminus \mathbb{Q}$ and $r_A \neq r_B$ if $A, B \in \mathcal{A}$ are distinct.
  - iff there is a continuous $F : \Psi_{\mathcal{A}} \to \mathbb{R}$ such that $F|\{x_A : A \in \mathcal{A}\}$ is injective.
- $\mathcal{A}$ is called *Q-family*
  - iff every subfamily $\mathcal{B} \subseteq \mathcal{A}$ can be separated from its complement $\mathcal{A} \setminus \mathcal{B}$.
  - iff $\Psi_{\mathcal{A}}$ is normal.
- $\mathcal{A}$ is *Mrówka's family* if $\Psi^* = \beta\Psi_{\mathcal{A}} \setminus \Psi_{\mathcal{A}}$ consist just of one point.
- $\mathcal{A}$ is *Lusin's family* if no two uncountable subfamilies of $\mathcal{A}$ can be separated.

- M. Hrušák, *Almost disjoint families and topology*. Recent progress in general topology. III, 601-638, 2014.
- F. Hernández-Hernández, M. Hrušák, *Topology of Mrówka-Isbell spaces*. Pseudocompact topological spaces, 253–289, Dev. Math., 55, 2018.

Outline of the talk

Outline of the talk

1. We consider an equivalent renorming of $C_0(\Psi_{\mathcal{A}})$ and obtain new results on the geometry of unit spheres of Banach spaces.

Outline of the talk

1. We consider an equivalent renorming of $C_0(\Psi_{\mathcal{A}})$ and obtain new results on the geometry of unit spheres of Banach spaces.

2. We consider $\mathbb{N} \times \mathbb{N}$ matrices acting on $C_0(\Psi_{\mathcal{A}})$ which generalize partitioners of almost disjont families or clopen subsets of $\Psi_{\mathcal{A}}$ and obtain new examples in Banach algebras of operators.

Outline of the talk

1. We consider an equivalent renorming of $C_0(\Psi_{\mathcal{A}})$ and obtain new results on the geometry of unit spheres of Banach spaces.

2. We consider $\mathbb{N} \times \mathbb{N}$ matrices acting on $C_0(\Psi_{\mathcal{A}})$ which generalize partitioners of almost disjont families or clopen subsets of $\Psi_{\mathcal{A}}$ and obtain new examples in Banach algebras of operators.

Outline of the talk

1. We consider an equivalent renorming of $C_0(\Psi_{\mathcal{A}})$ and obtain new results on the geometry of unit spheres of Banach spaces.

2. We consider $\mathbb{N} \times \mathbb{N}$ matrices acting on $C_0(\Psi_{\mathcal{A}})$ which generalize partitioners of almost disjont families or clopen subsets of $\Psi_{\mathcal{A}}$ and obtain new examples in Banach algebras of operators.

3. We consider continuous functions from $\Psi_{\mathcal{A}}$ into 2×2 matrices with pointwise noncommutative multiplication and obtain new results in the theory of C*-algebras .

Outline of the talk

1. We consider an equivalent renorming of $C_0(\Psi_{\mathcal{A}})$ and obtain new results on the geometry of unit spheres of Banach spaces.

2. We consider $\mathbb{N} \times \mathbb{N}$ matrices acting on $C_0(\Psi_{\mathcal{A}})$ which generalize partitioners of almost disjont families or clopen subsets of $\Psi_{\mathcal{A}}$ and obtain new examples in Banach algebras of operators.

3. We consider continuous functions from $\Psi_{\mathcal{A}}$ into 2×2 matrices with pointwise noncommutative multiplication and obtain new results in the theory of C*-algebras .

Outline of the talk

1. We consider an equivalent renorming of $C_0(\Psi_{\mathcal{A}})$ and obtain new results on the geometry of unit spheres of Banach spaces.

2. We consider $\mathbb{N} \times \mathbb{N}$ matrices acting on $C_0(\Psi_{\mathcal{A}})$ which generalize partitioners of almost disjont families or clopen subsets of $\Psi_{\mathcal{A}}$ and obtain new examples in Banach algebras of operators.

3. We consider continuous functions from $\Psi_{\mathcal{A}}$ into 2×2 matrices with pointwise noncommutative multiplication and obtain new results in the theory of C*-algebras
.

Outline of the talk

1. We consider an equivalent renorming of $C_0(\Psi_{\mathcal{A}})$ and obtain new results on the geometry of unit spheres of Banach spaces.
2. We consider $\mathbb{N} \times \mathbb{N}$ matrices acting on $C_0(\Psi_{\mathcal{A}})$ which generalize partitioners of almost disjont families or clopen subsets of $\Psi_{\mathcal{A}}$ and obtain new examples in Banach algebras of operators.
3. We consider continuous functions from $\Psi_{\mathcal{A}}$ into 2×2 matrices with pointwise noncommutative multiplication and obtain new results in the theory of C*-algebras .

In all theses cases we need extra combinatorial properties of the almost disjoint families to obtain interesting examples.

Subspaces of metric/Banach spaces

Subspaces of metric/Banach spaces

If $(\mathcal{X}, d)$ is a metric space, $\mathcal{Y} \subseteq \mathcal{X}$ and $r > 0$, we say that $\mathcal{Y}$ is

Subspaces of metric/Banach spaces

If $(\mathcal{X}, d)$ is a metric space, $\mathcal{Y} \subseteq \mathcal{X}$ and $r > 0$, we say that $\mathcal{Y}$ is

- $(r+)$-separated if $d(y, y') > r$, for any two distinct $y, y' \in \mathcal{Y}$,

Subspaces of metric/Banach spaces

If $(\mathcal{X}, d)$ is a metric space, $\mathcal{Y} \subseteq \mathcal{X}$ and $r > 0$, we say that $\mathcal{Y}$ is

- $(r+)$-separated if $d(y, y') > r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-separated if $d(y, y') \geq r$, for any two distinct $y, y' \in \mathcal{Y}$,

Subspaces of metric/Banach spaces

If $(\mathcal{X}, d)$ is a metric space, $\mathcal{Y} \subseteq \mathcal{X}$ and $r > 0$, we say that $\mathcal{Y}$ is

- $(r+)$-separated if $d(y, y') > r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-separated if $d(y, y') \geq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-concentrated if $d(y, y') \leq r$, for any two distinct $y, y' \in \mathcal{Y}$,

Subspaces of metric/Banach spaces

If $(\mathcal{X}, d)$ is a metric space, $\mathcal{Y} \subseteq \mathcal{X}$ and $r > 0$, we say that $\mathcal{Y}$ is

- $(r+)$-separated if $d(y, y') > r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-separated if $d(y, y') \geq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-concentrated if $d(y, y') \leq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-equilateral if $d(y, y') = r$ for any two distinct $y, y' \in \mathcal{Y}$,

Subspaces of metric/Banach spaces

If $(\mathcal{X}, d)$ is a metric space, $\mathcal{Y} \subseteq \mathcal{X}$ and $r > 0$, we say that $\mathcal{Y}$ is

- $(r+)$-separated if $d(y, y') > r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-separated if $d(y, y') \geq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-concentrated if $d(y, y') \leq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-equilateral if $d(y, y') = r$ for any two distinct $y, y' \in \mathcal{Y}$,
- equilateral if it is r-equilateral for some $r > 0$.

Subspaces of metric/Banach spaces

If $(\mathcal{X}, d)$ is a metric space, $\mathcal{Y} \subseteq \mathcal{X}$ and $r > 0$, we say that $\mathcal{Y}$ is

- $(r+)$-separated if $d(y, y') > r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-separated if $d(y, y') \geq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-concentrated if $d(y, y') \leq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-equilateral if $d(y, y') = r$ for any two distinct $y, y' \in \mathcal{Y}$,
- equilateral if it is r-equilateral for some $r > 0$.

Subspaces of metric/Banach spaces

If $(\mathcal{X}, d)$ is a metric space, $\mathcal{Y} \subseteq \mathcal{X}$ and $r > 0$, we say that $\mathcal{Y}$ is

- $(r+)$-separated if $d(y, y') > r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-separated if $d(y, y') \geq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-concentrated if $d(y, y') \leq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-equilateral if $d(y, y') = r$ for any two distinct $y, y' \in \mathcal{Y}$,
- equilateral if it is r-equilateral for some $r > 0$.

Let $(\mathcal{X}, \| \|)$ be an infinite dimensional Banach space and $S_{\mathcal{X}}$ its unit sphere.

Subspaces of metric/Banach spaces

If $(\mathcal{X}, d)$ is a metric space, $\mathcal{Y} \subseteq \mathcal{X}$ and $r > 0$, we say that $\mathcal{Y}$ is

- $(r+)$-separated if $d(y, y') > r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-separated if $d(y, y') \geq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-concentrated if $d(y, y') \leq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-equilateral if $d(y, y') = r$ for any two distinct $y, y' \in \mathcal{Y}$,
- equilateral if it is r-equilateral for some $r > 0$.

Let $(\mathcal{X}, \| \ \|)$ be an infinite dimensional Banach space and $S_{\mathcal{X}}$ its unit sphere.

- (Riesz, 1916) For every $\varepsilon > 0$ there is $(1 - \varepsilon)$-separated $\mathcal{Y} \subseteq S_{\mathcal{X}}$ with $|\mathcal{Y}| = dens(\mathcal{X})$.

Subspaces of metric/Banach spaces

If $(\mathcal{X}, d)$ is a metric space, $\mathcal{Y} \subseteq \mathcal{X}$ and $r > 0$, we say that $\mathcal{Y}$ is

- $(r+)$-separated if $d(y, y') > r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-separated if $d(y, y') \geq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-concentrated if $d(y, y') \leq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-equilateral if $d(y, y') = r$ for any two distinct $y, y' \in \mathcal{Y}$,
- equilateral if it is r-equilateral for some $r > 0$.

Let $(\mathcal{X}, \| \ \|)$ be an infinite dimensional Banach space and $S_{\mathcal{X}}$ its unit sphere.

- (Riesz, 1916) For every $\varepsilon > 0$ there is $(1 - \varepsilon)$-separated $\mathcal{Y} \subseteq S_{\mathcal{X}}$ with $|\mathcal{Y}| = dens(\mathcal{X})$.
- (Kottman, 1975) There is an infinite $(1+)$-separated $\mathcal{Y} \subseteq S_{\mathcal{X}}$.

Subspaces of metric/Banach spaces

If $(\mathcal{X}, d)$ is a metric space, $\mathcal{Y} \subseteq \mathcal{X}$ and $r > 0$, we say that $\mathcal{Y}$ is

- $(r+)$-separated if $d(y, y') > r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-separated if $d(y, y') \geq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-concentrated if $d(y, y') \leq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-equilateral if $d(y, y') = r$ for any two distinct $y, y' \in \mathcal{Y}$,
- equilateral if it is r-equilateral for some $r > 0$.

Let $(\mathcal{X}, \| \|)$ be an infinite dimensional Banach space and $S_{\mathcal{X}}$ its unit sphere.

- (Riesz, 1916) For every $\varepsilon > 0$ there is $(1 - \varepsilon)$-separated $\mathcal{Y} \subseteq S_{\mathcal{X}}$ with $|\mathcal{Y}| = dens(\mathcal{X})$.
- (Kottman, 1975) There is an infinite $(1+)$-separated $\mathcal{Y} \subseteq S_{\mathcal{X}}$.
- (Elton, Odell, 1981) There is an infinite $(1 + \varepsilon)$-separated $\mathcal{Y} \subseteq S_{\mathcal{X}}$ for some $\varepsilon > 0$.

Subspaces of metric/Banach spaces

If $(\mathcal{X}, d)$ is a metric space, $\mathcal{Y} \subseteq \mathcal{X}$ and $r > 0$, we say that $\mathcal{Y}$ is

- $(r+)$-separated if $d(y, y') > r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-separated if $d(y, y') \geq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-concentrated if $d(y, y') \leq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-equilateral if $d(y, y') = r$ for any two distinct $y, y' \in \mathcal{Y}$,
- equilateral if it is r-equilateral for some $r > 0$.

Let $(\mathcal{X}, \| \, \|)$ be an infinite dimensional Banach space and $S_{\mathcal{X}}$ its unit sphere.

- (Riesz, 1916) For every $\varepsilon > 0$ there is $(1-\varepsilon)$-separated $\mathcal{Y} \subseteq S_{\mathcal{X}}$ with $|\mathcal{Y}| = dens(\mathcal{X})$.
- (Kottman, 1975) There is an infinite $(1+)$-separated $\mathcal{Y} \subseteq S_{\mathcal{X}}$.
- (Elton, Odell, 1981) There is an infinite $(1+\varepsilon)$-separated $\mathcal{Y} \subseteq S_{\mathcal{X}}$ for some $\varepsilon > 0$.
- $c_0(\omega_1)$ does not admit an uncountable $(1+\varepsilon)$-separated set.

Subspaces of metric/Banach spaces

If $(\mathcal{X}, d)$ is a metric space, $\mathcal{Y} \subseteq \mathcal{X}$ and $r > 0$, we say that $\mathcal{Y}$ is

- $(r+)$-separated if $d(y, y') > r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-separated if $d(y, y') \geq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-concentrated if $d(y, y') \leq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-equilateral if $d(y, y') = r$ for any two distinct $y, y' \in \mathcal{Y}$,
- equilateral if it is r-equilateral for some $r > 0$.

Let $(\mathcal{X}, \| \ \|)$ be an infinite dimensional Banach space and $S_{\mathcal{X}}$ its unit sphere.

- (Riesz, 1916) For every $\varepsilon > 0$ there is $(1-\varepsilon)$-separated $\mathcal{Y} \subseteq S_{\mathcal{X}}$ with $|\mathcal{Y}| = dens(\mathcal{X})$.
- (Kottman, 1975) There is an infinite $(1+)$-separated $\mathcal{Y} \subseteq S_{\mathcal{X}}$.
- (Elton, Odell, 1981) There is an infinite $(1+\varepsilon)$-separated $\mathcal{Y} \subseteq S_{\mathcal{X}}$ for some $\varepsilon > 0$.
- $c_0(\omega_1)$ does not admit an uncountable $(1+\varepsilon)$-separated set.
- (P. K., 2018) It is independent whether a nonseparable Banach space of the form $X = C(K)$ admits an uncountable equilateral set.

Subspaces of metric/Banach spaces

If $(\mathcal{X}, d)$ is a metric space, $\mathcal{Y} \subseteq \mathcal{X}$ and $r > 0$, we say that $\mathcal{Y}$ is

- $(r+)$-separated if $d(y, y') > r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-separated if $d(y, y') \geq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-concentrated if $d(y, y') \leq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-equilateral if $d(y, y') = r$ for any two distinct $y, y' \in \mathcal{Y}$,
- equilateral if it is r-equilateral for some $r > 0$.

Let $(\mathcal{X}, \| \|)$ be an infinite dimensional Banach space and $S_{\mathcal{X}}$ its unit sphere.

- (Riesz, 1916) For every $\varepsilon > 0$ there is $(1-\varepsilon)$-separated $\mathcal{Y} \subseteq S_{\mathcal{X}}$ with $|\mathcal{Y}| = dens(\mathcal{X})$.
- (Kottman, 1975) There is an infinite $(1+)$-separated $\mathcal{Y} \subseteq S_{\mathcal{X}}$.
- (Elton, Odell, 1981) There is an infinite $(1+\varepsilon)$-separated $\mathcal{Y} \subseteq S_{\mathcal{X}}$ for some $\varepsilon > 0$.
- $c_0(\omega_1)$ does not admit an uncountable $(1+\varepsilon)$-separated set.
- (P. K., 2018) It is independent whether a nonseparable Banach space of the form $X = C(K)$ admits an uncountable equilateral set.

Subspaces of metric/Banach spaces

If $(\mathcal{X}, d)$ is a metric space, $\mathcal{Y} \subseteq \mathcal{X}$ and $r > 0$, we say that $\mathcal{Y}$ is

- $(r+)$-separated if $d(y, y') > r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-separated if $d(y, y') \geq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-concentrated if $d(y, y') \leq r$, for any two distinct $y, y' \in \mathcal{Y}$,
- r-equilateral if $d(y, y') = r$ for any two distinct $y, y' \in \mathcal{Y}$,
- equilateral if it is r-equilateral for some $r > 0$.

Let $(\mathcal{X}, \| \, \|)$ be an infinite dimensional Banach space and $S_{\mathcal{X}}$ its unit sphere.

- (Riesz, 1916) For every $\varepsilon > 0$ there is $(1 - \varepsilon)$-separated $\mathcal{Y} \subseteq S_{\mathcal{X}}$ with $|\mathcal{Y}| = dens(\mathcal{X})$.
- (Kottman, 1975) There is an infinite $(1+)$-separated $\mathcal{Y} \subseteq S_{\mathcal{X}}$.
- (Elton, Odell, 1981) There is an infinite $(1 + \varepsilon)$-separated $\mathcal{Y} \subseteq S_{\mathcal{X}}$ for some $\varepsilon > 0$.
- $c_0(\omega_1)$ does not admit an uncountable $(1 + \varepsilon)$-separated set.
- (P. K., 2018) It is independent whether a nonseparable Banach space of the form $X = C(K)$ admits an uncountable equilateral set.

The first application.

The first application.

Theorem (P.K. 202?)

The first application.

Theorem (P.K. 202?)

There is a (strictly convex) Banach space of density continuum where

The first application.

Theorem (P.K. 202?)

There is a (strictly convex) Banach space of density continuum where

- *every uncountable subset of the unit sphere of regular cardinality κ includes a subset of the same cardinality which is $(1-\varepsilon)$-concentrated for some $\varepsilon > 0$.*

The first application.

Theorem (P.K. 202?)

There is a (strictly convex) Banach space of density continuum where

- *every uncountable subset of the unit sphere of regular cardinality κ includes a subset of the same cardinality which is $(1-\varepsilon)$-concentrated for some $\varepsilon > 0$.*

The first application.

Theorem (P.K. 202?)

There is a (strictly convex) Banach space of density continuum where

- *every uncountable subset of the unit sphere of regular cardinality κ includes a subset of the same cardinality which is $(1-\varepsilon)$-concentrated for some $\varepsilon > 0$.*
- *there is no uncountable equilateral set.*

The first application.

Theorem (P.K. 202?)

There is a (strictly convex) Banach space of density continuum where

- *every uncountable subset of the unit sphere of regular cardinality κ includes a subset of the same cardinality which is $(1-\varepsilon)$-concentrated for some $\varepsilon > 0$.*
- *there is no uncountable equilateral set.*

The first application.

Theorem (P.K. 202?)

There is a (strictly convex) Banach space of density continuum where

- *every uncountable subset of the unit sphere of regular cardinality* κ *includes a subset of the same cardinality which is* $(1-\varepsilon)$*-concentrated for some* $\varepsilon > 0$*.*
- *there is no uncountable equilateral set.*

Proof.

The first application.

Theorem (P.K. 202?)

There is a (strictly convex) Banach space of density continuum where

- *every uncountable subset of the unit sphere of regular cardinality κ includes a subset of the same cardinality which is $(1-\varepsilon)$-concentrated for some $\varepsilon > 0$.*
- *there is no uncountable equilateral set.*

Proof.

Let $\mathcal{A}$ be an $\mathbb{R}$-embeddable almost disjoint family of subsets of $\mathbb{N}$.

The first application.

Theorem (P.K. 202?)

There is a (strictly convex) Banach space of density continuum where

- *every uncountable subset of the unit sphere of regular cardinality κ includes a subset of the same cardinality which is $(1-\varepsilon)$-concentrated for some $\varepsilon > 0$.*
- *there is no uncountable equilateral set.*

Proof.

Let $\mathcal{A}$ be an $\mathbb{R}$-embeddable almost disjoint family of subsets of $\mathbb{N}$. Consider $\mathcal{X} = C_0(\Psi_{\mathcal{A}})$ with

The first application.

Theorem (P.K. 202?)

There is a (strictly convex) Banach space of density continuum where

- *every uncountable subset of the unit sphere of regular cardinality κ includes a subset of the same cardinality which is $(1-\varepsilon)$-concentrated for some $\varepsilon > 0$.*
- *there is no uncountable equilateral set.*

Proof.

Let $\mathcal{A}$ be an $\mathbb{R}$-embeddable almost disjoint family of subsets of $\mathbb{N}$. Consider $\mathcal{X} = C_0(\Psi_{\mathcal{A}})$ with

$$|||f||| = \sup\{|f(x)| : x \in \Psi_A\} + \sqrt{\sum_{n\in\mathbb{N}} \frac{f(n)^2}{2^n}}.$$

□

The first application.

Theorem (P.K. 202?)

There is a (strictly convex) Banach space of density continuum where

- *every uncountable subset of the unit sphere of regular cardinality κ includes a subset of the same cardinality which is $(1-\varepsilon)$-concentrated for some $\varepsilon > 0$.*
- *there is no uncountable equilateral set.*

Proof.

Let $\mathcal{A}$ be an $\mathbb{R}$-embeddable almost disjoint family of subsets of $\mathbb{N}$. Consider $\mathcal{X} = C_0(\Psi_{\mathcal{A}})$ with

$$|||f||| = \sup\{|f(x)| : x \in \Psi_A\} + \sqrt{\sum_{n\in\mathbb{N}} \frac{f(n)^2}{2^n}}.$$

□

Theorem (P.K. 202?)

The first application.

Theorem (P.K. 202?)

There is a (strictly convex) Banach space of density continuum where

- *every uncountable subset of the unit sphere of regular cardinality κ includes a subset of the same cardinality which is $(1-\varepsilon)$-concentrated for some $\varepsilon > 0$.*
- *there is no uncountable equilateral set.*

Proof.

Let $\mathcal{A}$ be an $\mathbb{R}$-embeddable almost disjoint family of subsets of $\mathbb{N}$. Consider $\mathcal{X} = C_0(\Psi_{\mathcal{A}})$ with

$$|||f||| = \sup\{|f(x)| : x \in \Psi_A\} + \sqrt{\sum_{n\in\mathbb{N}} \frac{f(n)^2}{2^n}}.$$

□

Theorem (P.K. 202?)

Let $\mathcal{A}$ is an $\mathbb{R}$-embeddable almost disjoint family of subsets of $\mathbb{N}$ of cardinality continuum. Then $\Psi_{\mathcal{A}}$ is of weight continuum and such that

The first application.

Theorem (P.K. 202?)

There is a (strictly convex) Banach space of density continuum where

- *every uncountable subset of the unit sphere of regular cardinality κ includes a subset of the same cardinality which is $(1-\varepsilon)$-concentrated for some $\varepsilon > 0$.*
- *there is no uncountable equilateral set.*

Proof.

Let $\mathcal{A}$ be an $\mathbb{R}$-embeddable almost disjoint family of subsets of $\mathbb{N}$. Consider $\mathcal{X} = C_0(\Psi_{\mathcal{A}})$ with

$$|||f||| = \sup\{|f(x)| : x \in \Psi_A\} + \sqrt{\sum_{n\in\mathbb{N}} \frac{f(n)^2}{2^n}}.$$

□

Theorem (P.K. 202?)

Let $\mathcal{A}$ is an $\mathbb{R}$-embeddable almost disjoint family of subsets of $\mathbb{N}$ of cardinality continuum. Then $\Psi_{\mathcal{A}}$ is of weight continuum and such that for every $\varepsilon > 0$ and every subset $\mathcal{Y}$ of the unit sphere of $C_0(\Psi_{\mathcal{A}})$ of a regular uncountable cardinality

The first application.

Theorem (P.K. 202?)

There is a (strictly convex) Banach space of density continuum where

- *every uncountable subset of the unit sphere of regular cardinality κ includes a subset of the same cardinality which is $(1-\varepsilon)$-concentrated for some $\varepsilon > 0$.*
- *there is no uncountable equilateral set.*

Proof.

Let $\mathcal{A}$ be an $\mathbb{R}$-embeddable almost disjoint family of subsets of $\mathbb{N}$. Consider $\mathcal{X} = C_0(\Psi_\mathcal{A})$ with

$$|||f||| = \sup\{|f(x)| : x \in \Psi_A\} + \sqrt{\sum_{n\in\mathbb{N}} \frac{f(n)^2}{2^n}}.$$

□

Theorem (P.K. 202?)

Let $\mathcal{A}$ is an $\mathbb{R}$-embeddable almost disjoint family of subsets of $\mathbb{N}$ of cardinality continuum. Then $\Psi_\mathcal{A}$ is of weight continuum and such that for every $\varepsilon > 0$ and every subset $\mathcal{Y}$ of the unit sphere of $C_0(\Psi_\mathcal{A})$ of a regular uncountable cardinality there is a subset $\mathcal{Z} \subseteq \mathcal{Y}$ of the same cardinality which is $(1+\varepsilon)$-concentrated.

The first application.

Theorem (P.K. 202?)

There is a (strictly convex) Banach space of density continuum where

- *every uncountable subset of the unit sphere of regular cardinality κ includes a subset of the same cardinality which is $(1-\varepsilon)$-concentrated for some $\varepsilon > 0$.*
- *there is no uncountable equilateral set.*

Proof.

Let $\mathcal{A}$ be an $\mathbb{R}$-embeddable almost disjoint family of subsets of $\mathbb{N}$. Consider $\mathcal{X} = C_0(\Psi_{\mathcal{A}})$ with

$$|||f||| = \sup\{|f(x)| : x \in \Psi_A\} + \sqrt{\sum_{n\in\mathbb{N}} \frac{f(n)^2}{2^n}}.$$

□

Theorem (P.K. 202?)

Let $\mathcal{A}$ is an $\mathbb{R}$-embeddable almost disjoint family of subsets of $\mathbb{N}$ of cardinality continuum. Then $\Psi_{\mathcal{A}}$ is of weight continuum and such that for every $\varepsilon > 0$ and every subset $\mathcal{Y}$ of the unit sphere of $C_0(\Psi_{\mathcal{A}})$ of a regular uncountable cardinality there is a subset $\mathcal{Z} \subseteq \mathcal{Y}$ of the same cardinality which is $(1+\varepsilon)$-concentrated.

Questions

Questions

Question:

Questions

Question: Does every Banach space of density bigger than 2^ω admit an uncountable set which is $(1+)$-separated/equilateral?

Questions

Question: Does every Banach space of density bigger than 2^ω admit an uncountable set which is $(1+)$-separated/equilateral?

- (Terenzi, 1989) Every Banach space of density $> 2^{2^\omega}$ admits an uncountable equilateral set.

Questions

Question: Does every Banach space of density bigger than 2^ω admit an uncountable set which is $(1+)$-separated/equilateral?

- (Terenzi, 1989) Every Banach space of density $> 2^{2^\omega}$ admits an uncountable equilateral set.
- (Hájek, Kania, Russo, 2020) Every Banach space of density $> 2^{2^{2^{2^\omega}}}$ admits an uncountable $(1+)$-separated set.

Questions

Question: Does every Banach space of density bigger than 2^ω admit an uncountable set which is $(1+)$-separated/equilateral?

- (Terenzi, 1989) Every Banach space of density $> 2^{2^\omega}$ admits an uncountable equilateral set.
- (Hájek, Kania, Russo, 2020) Every Banach space of density $> 2^{2^{2^{2^\omega}}}$ admits an uncountable $(1+)$-separated set.

Questions

Question: Does every Banach space of density bigger than 2^ω admit an uncountable set which is $(1+)$-separated/equilateral?

- (Terenzi, 1989) Every Banach space of density $> 2^{2^\omega}$ admits an uncountable equilateral set.
- (Hájek, Kania, Russo, 2020) Every Banach space of density $> 2^{2^{2^{2^\omega}}}$ admits an uncountable $(1+)$-separated set.

Question:

Questions

Question: Does every Banach space of density bigger than 2^ω admit an uncountable set which is $(1+)$-separated/equilateral?

- (Terenzi, 1989) Every Banach space of density $> 2^{2^\omega}$ admits an uncountable equilateral set.
- (Hájek, Kania, Russo, 2020) Every Banach space of density $> 2^{2^{2^{2^\omega}}}$ admits an uncountable $(1+)$-separated set.

Question: Which metric spaces can be isometrically embedded into any infinite dimensional Banach space (of a given class)?

Questions

Question: Does every Banach space of density bigger than 2^ω admit an uncountable set which is $(1+)$-separated/equilateral?

- (Terenzi, 1989) Every Banach space of density $> 2^{2^\omega}$ admits an uncountable equilateral set.
- (Hájek, Kania, Russo, 2020) Every Banach space of density $> 2^{2^{2^{2^\omega}}}$ admits an uncountable $(1+)$-separated set.

Question: Which metric spaces can be isometrically embedded into any infinite dimensional Banach space (of a given class)?

- (Shkarin, 2004) Every infinite dimensional Banach space contains any finite ultrametric space.

Questions

Question: Does every Banach space of density bigger than 2^ω admit an uncountable set which is $(1+)$-separated/equilateral?

- (Terenzi, 1989) Every Banach space of density $> 2^{2^\omega}$ admits an uncountable equilateral set.
- (Hájek, Kania, Russo, 2020) Every Banach space of density $> 2^{2^{2^{2^\omega}}}$ admits an uncountable $(1+)$-separated set.

Question: Which metric spaces can be isometrically embedded into any infinite dimensional Banach space (of a given class)?

- (Shkarin, 2004) Every infinite dimensional Banach space contains any finite ultrametric space.

Questions

Question: Does every Banach space of density bigger than 2^ω admit an uncountable set which is $(1+)$-separated/equilateral?

- (Terenzi, 1989) Every Banach space of density $> 2^{2^\omega}$ admits an uncountable equilateral set.
- (Hájek, Kania, Russo, 2020) Every Banach space of density $> 2^{2^{2^{2^\omega}}}$ admits an uncountable $(1+)$-separated set.

Question: Which metric spaces can be isometrically embedded into any infinite dimensional Banach space (of a given class)?

- (Shkarin, 2004) Every infinite dimensional Banach space contains any finite ultrametric space.
- (Mercourakis, Vassiliadis 2014) If $\mathcal{X}$ admits an isomorphic copy of c_0, then $\mathcal{X}$ contains an infinite equilateral set.

Questions

Question: Does every Banach space of density bigger than 2^ω admit an uncountable set which is $(1+)$-separated/equilateral?

- (Terenzi, 1989) Every Banach space of density $> 2^{2^\omega}$ admits an uncountable equilateral set.
- (Hájek, Kania, Russo, 2020) Every Banach space of density $> 2^{2^{2^{2^\omega}}}$ admits an uncountable $(1+)$-separated set.

Question: Which metric spaces can be isometrically embedded into any infinite dimensional Banach space (of a given class)?

- (Shkarin, 2004) Every infinite dimensional Banach space contains any finite ultrametric space.
- (Mercourakis, Vassiliadis 2014) If $\mathcal{X}$ admits an isomorphic copy of c_0, then $\mathcal{X}$ contains an infinite equilateral set.

Questions

Question: Does every Banach space of density bigger than 2^ω admit an uncountable set which is $(1+)$-separated/equilateral?

- (Terenzi, 1989) Every Banach space of density $> 2^{2^\omega}$ admits an uncountable equilateral set.
- (Hájek, Kania, Russo, 2020) Every Banach space of density $> 2^{2^{2^{2^\omega}}}$ admits an uncountable $(1+)$-separated set.

Question: Which metric spaces can be isometrically embedded into any infinite dimensional Banach space (of a given class)?

- (Shkarin, 2004) Every infinite dimensional Banach space contains any finite ultrametric space.
- (Mercourakis, Vassiliadis 2014) If $\mathcal{X}$ admits an isomorphic copy of c_0, then $\mathcal{X}$ contains an infinite equilateral set.

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.
- If $g \in C_b(\Psi_\mathcal{A})$, then $M_g \in \mathcal{B}(C_0(\Psi_\mathcal{A}))$, where $M_g(f) = gf$ for $f \in C_0(\Psi_\mathcal{A})$.

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.
- If $g \in C_b(\Psi_{\mathcal{A}})$, then $M_g \in \mathcal{B}(C_0(\Psi_{\mathcal{A}}))$, where $M_g(f) = gf$ for $f \in C_0(\Psi_{\mathcal{A}})$.
- $\mathcal{M}(C_0(\Psi_{\mathcal{A}})) = \{M_g : g \in C_b(\Psi_{\mathcal{A}})\}$ - the algebra of multipliers of $C_0(\Psi_{\mathcal{A}})$.

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.
- If $g \in C_b(\Psi_\mathcal{A})$, then $M_g \in \mathcal{B}(C_0(\Psi_\mathcal{A}))$, where $M_g(f) = gf$ for $f \in C_0(\Psi_\mathcal{A})$.
- $\mathcal{M}(C_0(\Psi_\mathcal{A})) = \{M_g : g \in C_b(\Psi_\mathcal{A})\}$ - the algebra of multipliers of $C_0(\Psi_\mathcal{A})$.
- If $\mathcal{A}$ is maximal, then the multiplier M_g has separable range iff $g \in C_0(\Psi_\mathcal{A})$, so

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.
- If $g \in C_b(\Psi_{\mathcal{A}})$, then $M_g \in \mathcal{B}(C_0(\Psi_{\mathcal{A}}))$, where $M_g(f) = gf$ for $f \in C_0(\Psi_{\mathcal{A}})$.
- $\mathcal{M}(C_0(\Psi_{\mathcal{A}})) = \{M_g : g \in C_b(\Psi_{\mathcal{A}})\}$ - the algebra of multipliers of $C_0(\Psi_{\mathcal{A}})$.
- If $\mathcal{A}$ is maximal, then the multiplier M_g has separable range iff $g \in C_0(\Psi_{\mathcal{A}})$, so

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.
- If $g \in C_b(\Psi_\mathcal{A})$, then $M_g \in \mathcal{B}(C_0(\Psi_\mathcal{A}))$, where $M_g(f) = gf$ for $f \in C_0(\Psi_\mathcal{A})$.
- $\mathcal{M}(C_0(\Psi_\mathcal{A})) = \{M_g : g \in C_b(\Psi_\mathcal{A})\}$ - the algebra of multipliers of $C_0(\Psi_\mathcal{A})$.
- If $\mathcal{A}$ is maximal, then the multiplier M_g has separable range iff $g \in C_0(\Psi_\mathcal{A})$, so
  $\mathcal{M}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(C_0(\Psi_A)) \equiv$

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.
- If $g \in C_b(\Psi_{\mathcal{A}})$, then $M_g \in \mathcal{B}(C_0(\Psi_{\mathcal{A}}))$, where $M_g(f) = gf$ for $f \in C_0(\Psi_{\mathcal{A}})$.
- $\mathcal{M}(C_0(\Psi_{\mathcal{A}})) = \{M_g : g \in C_b(\Psi_{\mathcal{A}})\}$ - the algebra of multipliers of $C_0(\Psi_{\mathcal{A}})$.
- If $\mathcal{A}$ is maximal, then the multiplier M_g has separable range iff $g \in C_0(\Psi_{\mathcal{A}})$, so
  $\mathcal{M}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(C_0(\Psi_A)) \equiv C_b(\Psi_{\mathcal{A}})/C_0(\Psi_{\mathcal{A}}) \equiv$

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
    - compact operators - $\mathcal{K}(\mathcal{X})$,
    - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
    - separable range operators - $\mathcal{S}(\mathcal{X})$.
- If $g \in C_b(\Psi_{\mathcal{A}})$, then $M_g \in \mathcal{B}(C_0(\Psi_{\mathcal{A}}))$, where $M_g(f) = gf$ for $f \in C_0(\Psi_{\mathcal{A}})$.
- $\mathcal{M}(C_0(\Psi_{\mathcal{A}})) = \{M_g : g \in C_b(\Psi_{\mathcal{A}})\}$ - the algebra of multipliers of $C_0(\Psi_{\mathcal{A}})$.
- If $\mathcal{A}$ is maximal, then the multiplier M_g has separable range iff $g \in C_0(\Psi_{\mathcal{A}})$, so
  $\mathcal{M}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(C_0(\Psi_A)) \equiv C_b(\Psi_{\mathcal{A}})/C_0(\Psi_{\mathcal{A}}) \equiv C(\beta\Psi_{\mathcal{A}} \setminus \Psi_{\mathcal{A}}) \equiv$

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.
- If $g \in C_b(\Psi_{\mathcal{A}})$, then $M_g \in \mathcal{B}(C_0(\Psi_{\mathcal{A}}))$, where $M_g(f) = gf$ for $f \in C_0(\Psi_{\mathcal{A}})$.
- $\mathcal{M}(C_0(\Psi_{\mathcal{A}})) = \{M_g : g \in C_b(\Psi_{\mathcal{A}})\}$ - the algebra of multipliers of $C_0(\Psi_{\mathcal{A}})$.
- If $\mathcal{A}$ is maximal, then the multiplier M_g has separable range iff $g \in C_0(\Psi_{\mathcal{A}})$, so
  $$\mathcal{M}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(C_0(\Psi_A)) \equiv C_b(\Psi_{\mathcal{A}})/C_0(\Psi_{\mathcal{A}}) \equiv C(\beta\Psi_{\mathcal{A}} \setminus \Psi_{\mathcal{A}}) \equiv C(\Psi_{\mathcal{A}}^*)$$

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.
- If $g \in C_b(\Psi_{\mathcal{A}})$, then $M_g \in \mathcal{B}(C_0(\Psi_{\mathcal{A}}))$, where $M_g(f) = gf$ for $f \in C_0(\Psi_{\mathcal{A}})$.
- $\mathcal{M}(C_0(\Psi_{\mathcal{A}})) = \{M_g : g \in C_b(\Psi_{\mathcal{A}})\}$ - the algebra of multipliers of $C_0(\Psi_{\mathcal{A}})$.
- If $\mathcal{A}$ is maximal, then the multiplier M_g has separable range iff $g \in C_0(\Psi_{\mathcal{A}})$, so
$$\mathcal{M}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(C_0(\Psi_A)) \equiv C_b(\Psi_{\mathcal{A}})/C_0(\Psi_{\mathcal{A}}) \equiv C(\beta\Psi_{\mathcal{A}} \setminus \Psi_{\mathcal{A}}) \equiv C(\Psi_{\mathcal{A}}^*)$$
- Let Ψ^* be the class of all Čech-Stone reminders of $\Psi_{\mathcal{A}}$s for maximal $\mathcal{A}$s

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.
- If $g \in C_b(\Psi_\mathcal{A})$, then $M_g \in \mathcal{B}(C_0(\Psi_\mathcal{A}))$, where $M_g(f) = gf$ for $f \in C_0(\Psi_\mathcal{A})$.
- $\mathcal{M}(C_0(\Psi_\mathcal{A})) = \{M_g : g \in C_b(\Psi_\mathcal{A})\}$ - the algebra of multipliers of $C_0(\Psi_\mathcal{A})$.
- If $\mathcal{A}$ is maximal, then the multiplier M_g has separable range iff $g \in C_0(\Psi_\mathcal{A})$, so
  $$\mathcal{M}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(C_0(\Psi_A)) \equiv C_b(\Psi_\mathcal{A})/C_0(\Psi_\mathcal{A}) \equiv C(\beta\Psi_\mathcal{A} \setminus \Psi_\mathcal{A}) \equiv C(\Psi^*_\mathcal{A})$$
- Let Ψ^* be the class of all Čech-Stone reminders of $\Psi_\mathcal{A}$s for maximal $\mathcal{A}$s

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.
- If $g \in C_b(\Psi_\mathcal{A})$, then $M_g \in \mathcal{B}(C_0(\Psi_\mathcal{A}))$, where $M_g(f) = gf$ for $f \in C_0(\Psi_\mathcal{A})$.
- $\mathcal{M}(C_0(\Psi_\mathcal{A})) = \{M_g : g \in C_b(\Psi_\mathcal{A})\}$ - the algebra of multipliers of $C_0(\Psi_\mathcal{A})$.
- If $\mathcal{A}$ is maximal, then the multiplier M_g has separable range iff $g \in C_0(\Psi_\mathcal{A})$, so
$$\mathcal{M}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(C_0(\Psi_A)) \equiv C_b(\Psi_\mathcal{A})/C_0(\Psi_\mathcal{A}) \equiv C(\beta\Psi_\mathcal{A} \setminus \Psi_\mathcal{A}) \equiv C(\Psi^*_\mathcal{A})$$
- Let Ψ^* be the class of all Čech-Stone reminders of $\Psi_\mathcal{A}$s for maximal $\mathcal{A}$s
  - (Terasawa, 1980) $\{$all compact metrizable spaces$\} \subseteq \Psi^*$.

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.
- If $g \in C_b(\Psi_{\mathcal{A}})$, then $M_g \in \mathcal{B}(C_0(\Psi_{\mathcal{A}}))$, where $M_g(f) = gf$ for $f \in C_0(\Psi_{\mathcal{A}})$.
- $\mathcal{M}(C_0(\Psi_{\mathcal{A}})) = \{M_g : g \in C_b(\Psi_{\mathcal{A}})\}$ - the algebra of multipliers of $C_0(\Psi_{\mathcal{A}})$.
- If $\mathcal{A}$ is maximal, then the multiplier M_g has separable range iff $g \in C_0(\Psi_{\mathcal{A}})$, so
$$\mathcal{M}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(C_0(\Psi_A)) \equiv C_b(\Psi_{\mathcal{A}})/C_0(\Psi_{\mathcal{A}}) \equiv C(\beta\Psi_{\mathcal{A}} \setminus \Psi_{\mathcal{A}}) \equiv C(\Psi^*_{\mathcal{A}})$$
- Let Ψ^* be the class of all Čech-Stone reminders of $\Psi_{\mathcal{A}}$s for maximal $\mathcal{A}$s
  - (Terasawa, 1980) $\{$all compact metrizable spaces$\} \subseteq \Psi^*$.

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.
- If $g \in C_b(\Psi_{\mathcal{A}})$, then $M_g \in \mathcal{B}(C_0(\Psi_{\mathcal{A}}))$, where $M_g(f) = gf$ for $f \in C_0(\Psi_{\mathcal{A}})$.
- $\mathcal{M}(C_0(\Psi_{\mathcal{A}})) = \{M_g : g \in C_b(\Psi_{\mathcal{A}})\}$ - the algebra of multipliers of $C_0(\Psi_{\mathcal{A}})$.
- If $\mathcal{A}$ is maximal, then the multiplier M_g has separable range iff $g \in C_0(\Psi_{\mathcal{A}})$, so
$$\mathcal{M}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(C_0(\Psi_A)) \equiv C_b(\Psi_{\mathcal{A}})/C_0(\Psi_{\mathcal{A}}) \equiv C(\beta\Psi_{\mathcal{A}} \setminus \Psi_{\mathcal{A}}) \equiv C(\Psi^*_{\mathcal{A}})$$
- Let Ψ^* be the class of all Čech-Stone reminders of $\Psi_{\mathcal{A}}$s for maximal $\mathcal{A}$s
  - (Terasawa, 1980) $\{$all compact metrizable spaces$\} \subseteq \Psi^*$.
  - (Baumgartner, Weese, 1982) (CH) $\Psi^* = \{$all compact spaces of weight $2^\omega\}$.

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.
- If $g \in C_b(\Psi_\mathcal{A})$, then $M_g \in \mathcal{B}(C_0(\Psi_\mathcal{A}))$, where $M_g(f) = gf$ for $f \in C_0(\Psi_\mathcal{A})$.
- $\mathcal{M}(C_0(\Psi_\mathcal{A})) = \{M_g : g \in C_b(\Psi_\mathcal{A})\}$ - the algebra of multipliers of $C_0(\Psi_\mathcal{A})$.
- If $\mathcal{A}$ is maximal, then the multiplier M_g has separable range iff $g \in C_0(\Psi_\mathcal{A})$, so
$$\mathcal{M}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(C_0(\Psi_A)) \equiv C_b(\Psi_\mathcal{A})/C_0(\Psi_\mathcal{A}) \equiv C(\beta\Psi_\mathcal{A} \setminus \Psi_\mathcal{A}) \equiv C(\Psi_\mathcal{A}^*)$$
- Let Ψ^* be the class of all Čech-Stone reminders of $\Psi_\mathcal{A}$s for maximal $\mathcal{A}$s
  - (Terasawa, 1980) $\{$all compact metrizable spaces$\} \subseteq \Psi^*$.
  - (Baumgartner, Weese, 1982) (CH) $\Psi^* = \{$all compact spaces of weight $2^\omega\}$.

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.
- If $g \in C_b(\Psi_\mathcal{A})$, then $M_g \in \mathcal{B}(C_0(\Psi_\mathcal{A}))$, where $M_g(f) = gf$ for $f \in C_0(\Psi_\mathcal{A})$.
- $\mathcal{M}(C_0(\Psi_\mathcal{A})) = \{M_g : g \in C_b(\Psi_\mathcal{A})\}$ - the algebra of multipliers of $C_0(\Psi_\mathcal{A})$.
- If $\mathcal{A}$ is maximal, then the multiplier M_g has separable range iff $g \in C_0(\Psi_\mathcal{A})$, so
  $$\mathcal{M}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(C_0(\Psi_A)) \equiv C_b(\Psi_\mathcal{A})/C_0(\Psi_\mathcal{A}) \equiv C(\beta\Psi_\mathcal{A} \setminus \Psi_\mathcal{A}) \equiv C(\Psi^*_\mathcal{A})$$
- Let Ψ^* be the class of all Čech-Stone reminders of $\Psi_\mathcal{A}$s for maximal $\mathcal{A}$s
  - (Terasawa, 1980) $\{$all compact metrizable spaces$\} \subseteq \Psi^*$.
  - (Baumgartner, Weese, 1982) (CH) $\Psi^* = \{$all compact spaces of weight $2^\omega\}$.
  - (Dow, 1990) $\Psi^* \subseteq \{$all compact spaces of cardinality$\leq 2^\omega\}$ $\not\ni \{0,1\}^\mathbb{R}$.

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.
- If $g \in C_b(\Psi_{\mathcal{A}})$, then $M_g \in \mathcal{B}(C_0(\Psi_{\mathcal{A}}))$, where $M_g(f) = gf$ for $f \in C_0(\Psi_{\mathcal{A}})$.
- $\mathcal{M}(C_0(\Psi_{\mathcal{A}})) = \{M_g : g \in C_b(\Psi_{\mathcal{A}})\}$ - the algebra of multipliers of $C_0(\Psi_{\mathcal{A}})$.
- If $\mathcal{A}$ is maximal, then the multiplier M_g has separable range iff $g \in C_0(\Psi_{\mathcal{A}})$, so
  $$\mathcal{M}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(C_0(\Psi_A)) \equiv C_b(\Psi_{\mathcal{A}})/C_0(\Psi_{\mathcal{A}}) \equiv C(\beta\Psi_{\mathcal{A}} \setminus \Psi_{\mathcal{A}}) \equiv C(\Psi^*_{\mathcal{A}})$$
- Let Ψ^* be the class of all Čech-Stone reminders of $\Psi_{\mathcal{A}}$s for maximal $\mathcal{A}$s
  - (Terasawa, 1980) $\{$all compact metrizable spaces$\} \subseteq \Psi^*$.
  - (Baumgartner, Weese, 1982) (CH) $\Psi^* = \{$all compact spaces of weight $2^\omega\}$.
  - (Dow, 1990) $\Psi^* \subseteq \{$all compact spaces of cardinality$\leq 2^\omega\} \not\ni \{0,1\}^{\mathbb{R}}$.
- What Banach algebras could be $\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A))$?

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.
- If $g \in C_b(\Psi_\mathcal{A})$, then $M_g \in \mathcal{B}(C_0(\Psi_\mathcal{A}))$, where $M_g(f) = gf$ for $f \in C_0(\Psi_\mathcal{A})$.
- $\mathcal{M}(C_0(\Psi_\mathcal{A})) = \{M_g : g \in C_b(\Psi_\mathcal{A})\}$ - the algebra of multipliers of $C_0(\Psi_\mathcal{A})$.
- If $\mathcal{A}$ is maximal, then the multiplier M_g has separable range iff $g \in C_0(\Psi_\mathcal{A})$, so $$\mathcal{M}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(C_0(\Psi_A)) \equiv C_b(\Psi_\mathcal{A})/C_0(\Psi_\mathcal{A}) \equiv C(\beta\Psi_\mathcal{A} \setminus \Psi_\mathcal{A}) \equiv C(\Psi^*_\mathcal{A})$$
- Let Ψ^* be the class of all Čech-Stone reminders of $\Psi_\mathcal{A}$s for maximal $\mathcal{A}$s
  - (Terasawa, 1980) $\{$all compact metrizable spaces$\} \subseteq \Psi^*$.
  - (Baumgartner, Weese, 1982) (CH) $\Psi^* = \{$all compact spaces of weight $2^\omega\}$.
  - (Dow, 1990) $\Psi^* \subseteq \{$all compact spaces of cardinality $\leq 2^\omega\} \not\ni \{0,1\}^\mathbb{R}$.
- What Banach algebras could be $\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A))$?

Banach algebras $\mathcal{B}(\mathcal{X})/\mathcal{I}$

- We consider the Banach algebra $\mathcal{B}(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the quotients $\mathcal{B}(\mathcal{X})/\mathcal{I}$ for classical ideals $\mathcal{I}$ like
  - compact operators - $\mathcal{K}(\mathcal{X})$,
  - weakly compact operators - $\mathcal{WC}(\mathcal{X})$,
  - separable range operators - $\mathcal{S}(\mathcal{X})$.
- If $g \in C_b(\Psi_{\mathcal{A}})$, then $M_g \in \mathcal{B}(C_0(\Psi_{\mathcal{A}}))$, where $M_g(f) = gf$ for $f \in C_0(\Psi_{\mathcal{A}})$.
- $\mathcal{M}(C_0(\Psi_{\mathcal{A}})) = \{M_g : g \in C_b(\Psi_{\mathcal{A}})\}$ - the algebra of multipliers of $C_0(\Psi_{\mathcal{A}})$.
- If $\mathcal{A}$ is maximal, then the multiplier M_g has separable range iff $g \in C_0(\Psi_{\mathcal{A}})$, so
  $$\mathcal{M}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(C_0(\Psi_A)) \equiv C_b(\Psi_{\mathcal{A}})/C_0(\Psi_{\mathcal{A}}) \equiv C(\beta\Psi_{\mathcal{A}} \setminus \Psi_{\mathcal{A}}) \equiv C(\Psi^*_{\mathcal{A}})$$
- Let Ψ^* be the class of all Čech-Stone reminders of $\Psi_{\mathcal{A}}$s for maximal $\mathcal{A}$s
  - (Terasawa, 1980) $\{$all compact metrizable spaces$\} \subseteq \Psi^*$.
  - (Baumgartner, Weese, 1982) (CH) $\Psi^* = \{$all compact spaces of weight $2^\omega\}$.
  - (Dow, 1990) $\Psi^* \subseteq \{$all compact spaces of cardinality$\leq 2^\omega\} \not\ni \{0,1\}^{\mathbb{R}}$.
- What Banach algebras could be $\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A))$?

The second application

The second application

Theorem (P.K., N. Laustsen, 2021)

The second application

Theorem (P.K., N. Laustsen, 2021)

There is an uncountable, almost disjoint family $\mathcal{A}$ such that:

The second application

Theorem (P.K., N. Laustsen, 2021)

There is an uncountable, almost disjoint family $\mathcal{A}$ such that:

- *every bounded linear operator $T : C_0(\Psi_{\mathcal{A}}) \to C_0(\Psi_{\mathcal{A}})$ has the form*

$$T = \lambda Id + S$$

  for some scalar λ and some operator $S : C_0(K_{\mathcal{A}}) \to C_0(K_{\mathcal{A}})$ that factors through c_0.

The second application

Theorem (P.K., N. Laustsen, 2021)

There is an uncountable, almost disjoint family $\mathcal{A}$ such that:

- *every bounded linear operator $T : C_0(\Psi_{\mathcal{A}}) \to C_0(\Psi_{\mathcal{A}})$ has the form*

$$T = \lambda Id + S$$

for some scalar λ and some operator $S : C_0(K_{\mathcal{A}}) \to C_0(K_{\mathcal{A}})$ that factors through c_0.

The second application

Theorem (P.K., N. Laustsen, 2021)

There is an uncountable, almost disjoint family $\mathcal{A}$ such that:

- *every bounded linear operator $T : C_0(\Psi_{\mathcal{A}}) \to C_0(\Psi_{\mathcal{A}})$ has the form*

$$T = \lambda Id + S$$

for some scalar λ and some operator $S : C_0(K_{\mathcal{A}}) \to C_0(K_{\mathcal{A}})$ that factors through c_0. In particular

$$\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A)) = \mathbb{R}, \mathbb{C}.$$

The second application

Theorem (P.K., N. Laustsen, 2021)

There is an uncountable, almost disjoint family $\mathcal{A}$ such that:

- *every bounded linear operator $T : C_0(\Psi_{\mathcal{A}}) \to C_0(\Psi_{\mathcal{A}})$ has the form*

$$T = \lambda Id + S$$

  for some scalar λ and some operator $S : C_0(K_{\mathcal{A}}) \to C_0(K_{\mathcal{A}})$ that factors through c_0. In particular

$$\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A)) = \mathbb{R}, \mathbb{C}.$$

- *Whenever the Banach space $C_0(K_{\mathcal{A}})$ is decomposed into a direct sum $C_0(K_{\mathcal{A}}) = \mathcal{X} \oplus \mathcal{Y}$ of two closed, infinite-dimensional subspaces $\mathcal{X}$ and $\mathcal{Y}$, then*

The second application

Theorem (P.K., N. Laustsen, 2021)

There is an uncountable, almost disjoint family $\mathcal{A}$ such that:

- *every bounded linear operator $T : C_0(\Psi_{\mathcal{A}}) \to C_0(\Psi_{\mathcal{A}})$ has the form*

$$T = \lambda Id + S$$

  for some scalar λ and some operator $S : C_0(K_{\mathcal{A}}) \to C_0(K_{\mathcal{A}})$ that factors through c_0. In particular

$$\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A)) = \mathbb{R}, \mathbb{C}.$$

- *Whenever the Banach space $C_0(K_{\mathcal{A}})$ is decomposed into a direct sum $C_0(K_{\mathcal{A}}) = \mathcal{X} \oplus \mathcal{Y}$ of two closed, infinite-dimensional subspaces $\mathcal{X}$ and $\mathcal{Y}$, then*

The second application

Theorem (P.K., N. Laustsen, 2021)

There is an uncountable, almost disjoint family $\mathcal{A}$ such that:

- *every bounded linear operator $T : C_0(\Psi_{\mathcal{A}}) \to C_0(\Psi_{\mathcal{A}})$ has the form*

$$T = \lambda Id + S$$

  for some scalar λ and some operator $S : C_0(K_{\mathcal{A}}) \to C_0(K_{\mathcal{A}})$ that factors through c_0. In particular

$$\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A)) = \mathbb{R}, \mathbb{C}.$$

- *Whenever the Banach space $C_0(K_{\mathcal{A}})$ is decomposed into a direct sum $C_0(K_{\mathcal{A}}) = \mathcal{X} \oplus \mathcal{Y}$ of two closed, infinite-dimensional subspaces $\mathcal{X}$ and $\mathcal{Y}$, then either $\mathcal{X}$ is isomorphic to $C_0(K_{\mathcal{A}})$ and $\mathcal{Y}$ is isomorphic to c_0,*

The second application

Theorem (P.K., N. Laustsen, 2021)

There is an uncountable, almost disjoint family $\mathcal{A}$ such that:

- *every bounded linear operator $T : C_0(\Psi_{\mathcal{A}}) \to C_0(\Psi_{\mathcal{A}})$ has the form*

$$T = \lambda Id + S$$

  for some scalar λ and some operator $S : C_0(K_{\mathcal{A}}) \to C_0(K_{\mathcal{A}})$ that factors through c_0. In particular

$$\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A)) = \mathbb{R}, \mathbb{C}.$$

- *Whenever the Banach space $C_0(K_{\mathcal{A}})$ is decomposed into a direct sum $C_0(K_{\mathcal{A}}) = \mathcal{X} \oplus \mathcal{Y}$ of two closed, infinite-dimensional subspaces $\mathcal{X}$ and $\mathcal{Y}$, then either $\mathcal{X}$ is isomorphic to $C_0(K_{\mathcal{A}})$ and $\mathcal{Y}$ is isomorphic to c_0, or vice versa.*

The second application

Theorem (P.K., N. Laustsen, 2021)

There is an uncountable, almost disjoint family $\mathcal{A}$ such that:

- *every bounded linear operator $T : C_0(\Psi_{\mathcal{A}}) \to C_0(\Psi_{\mathcal{A}})$ has the form*

$$T = \lambda Id + S$$

  for some scalar λ and some operator $S : C_0(K_{\mathcal{A}}) \to C_0(K_{\mathcal{A}})$ that factors through c_0. In particular

$$\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A)) = \mathbb{R}, \mathbb{C}.$$

- *Whenever the Banach space $C_0(K_{\mathcal{A}})$ is decomposed into a direct sum $C_0(K_{\mathcal{A}}) = \mathcal{X} \oplus \mathcal{Y}$ of two closed, infinite-dimensional subspaces $\mathcal{X}$ and $\mathcal{Y}$, then either $\mathcal{X}$ is isomorphic to $C_0(K_{\mathcal{A}})$ and $\mathcal{Y}$ is isomorphic to c_0, or vice versa.*

Questions:

The second application

Theorem (P.K., N. Laustsen, 2021)

There is an uncountable, almost disjoint family $\mathcal{A}$ such that:

- *every bounded linear operator $T : C_0(\Psi_{\mathcal{A}}) \to C_0(\Psi_{\mathcal{A}})$ has the form*

  $$T = \lambda Id + S$$

  for some scalar λ and some operator $S : C_0(K_{\mathcal{A}}) \to C_0(K_{\mathcal{A}})$ that factors through c_0. In particular

  $$\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A)) = \mathbb{R}, \mathbb{C}.$$

- *Whenever the Banach space $C_0(K_{\mathcal{A}})$ is decomposed into a direct sum $C_0(K_{\mathcal{A}}) = \mathcal{X} \oplus \mathcal{Y}$ of two closed, infinite-dimensional subspaces $\mathcal{X}$ and $\mathcal{Y}$, then either $\mathcal{X}$ is isomorphic to $C_0(K_{\mathcal{A}})$ and $\mathcal{Y}$ is isomorphic to c_0, or vice versa.*

Questions:

- What could be $\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A))$ other than $\mathbb{R}, \mathbb{C}$?

The second application

Theorem (P.K., N. Laustsen, 2021)

There is an uncountable, almost disjoint family $\mathcal{A}$ such that:

- *every bounded linear operator $T : C_0(\Psi_{\mathcal{A}}) \to C_0(\Psi_{\mathcal{A}})$ has the form*

$$T = \lambda Id + S$$

  for some scalar λ and some operator $S : C_0(K_{\mathcal{A}}) \to C_0(K_{\mathcal{A}})$ that factors through c_0. In particular

$$\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A)) = \mathbb{R}, \mathbb{C}.$$

- *Whenever the Banach space $C_0(K_{\mathcal{A}})$ is decomposed into a direct sum $C_0(K_{\mathcal{A}}) = \mathcal{X} \oplus \mathcal{Y}$ of two closed, infinite-dimensional subspaces $\mathcal{X}$ and $\mathcal{Y}$, then either $\mathcal{X}$ is isomorphic to $C_0(K_{\mathcal{A}})$ and $\mathcal{Y}$ is isomorphic to c_0, or vice versa.*

Questions:

- What could be $\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A))$ other than $\mathbb{R}, \mathbb{C}$?

The second application

Theorem (P.K., N. Laustsen, 2021)

There is an uncountable, almost disjoint family $\mathcal{A}$ such that:

- *every bounded linear operator $T : C_0(\Psi_{\mathcal{A}}) \to C_0(\Psi_{\mathcal{A}})$ has the form*

$$T = \lambda Id + S$$

  for some scalar λ and some operator $S : C_0(K_{\mathcal{A}}) \to C_0(K_{\mathcal{A}})$ that factors through c_0. In particular

$$\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A)) = \mathbb{R}, \mathbb{C}.$$

- *Whenever the Banach space $C_0(K_{\mathcal{A}})$ is decomposed into a direct sum $C_0(K_{\mathcal{A}}) = \mathcal{X} \oplus \mathcal{Y}$ of two closed, infinite-dimensional subspaces $\mathcal{X}$ and $\mathcal{Y}$, then either $\mathcal{X}$ is isomorphic to $C_0(K_{\mathcal{A}})$ and $\mathcal{Y}$ is isomorphic to c_0, or vice versa.*

Questions:

- What could be $\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A))$ other than $\mathbb{R}, \mathbb{C}$? $C(2^{\mathbb{N}})$?

The second application

Theorem (P.K., N. Laustsen, 2021)

There is an uncountable, almost disjoint family $\mathcal{A}$ such that:

- *every bounded linear operator $T : C_0(\Psi_{\mathcal{A}}) \to C_0(\Psi_{\mathcal{A}})$ has the form*

$$T = \lambda Id + S$$

  for some scalar λ and some operator $S : C_0(K_{\mathcal{A}}) \to C_0(K_{\mathcal{A}})$ that factors through c_0. In particular

$$\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A)) = \mathbb{R}, \mathbb{C}.$$

- *Whenever the Banach space $C_0(K_{\mathcal{A}})$ is decomposed into a direct sum $C_0(K_{\mathcal{A}}) = \mathcal{X} \oplus \mathcal{Y}$ of two closed, infinite-dimensional subspaces $\mathcal{X}$ and $\mathcal{Y}$, then either $\mathcal{X}$ is isomorphic to $C_0(K_{\mathcal{A}})$ and $\mathcal{Y}$ is isomorphic to c_0, or vice versa.*

Questions:

- What could be $\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A))$ other than $\mathbb{R}, \mathbb{C}$? $C(2^{\mathbb{N}})$? $C(2^{\mathbb{N}}, M_2)$?

The second application

Theorem (P.K., N. Laustsen, 2021)

There is an uncountable, almost disjoint family $\mathcal{A}$ such that:

- *every bounded linear operator $T : C_0(\Psi_{\mathcal{A}}) \to C_0(\Psi_{\mathcal{A}})$ has the form*

$$T = \lambda Id + S$$

  for some scalar λ and some operator $S : C_0(K_{\mathcal{A}}) \to C_0(K_{\mathcal{A}})$ that factors through c_0. In particular

$$\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A)) = \mathbb{R}, \mathbb{C}.$$

- *Whenever the Banach space $C_0(K_{\mathcal{A}})$ is decomposed into a direct sum $C_0(K_{\mathcal{A}}) = \mathcal{X} \oplus \mathcal{Y}$ of two closed, infinite-dimensional subspaces $\mathcal{X}$ and $\mathcal{Y}$, then either $\mathcal{X}$ is isomorphic to $C_0(K_{\mathcal{A}})$ and $\mathcal{Y}$ is isomorphic to c_0, or vice versa.*

Questions:

- What could be $\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A))$ other than $\mathbb{R}, \mathbb{C}$? $C(2^{\mathbb{N}})$? $C(2^{\mathbb{N}}, M_2)$?
- Can one construct in ZFC a maximal $\mathcal{A}$ as above?

The second application

Theorem (P.K., N. Laustsen, 2021)

There is an uncountable, almost disjoint family $\mathcal{A}$ such that:

- *every bounded linear operator $T : C_0(\Psi_{\mathcal{A}}) \to C_0(\Psi_{\mathcal{A}})$ has the form*

$$T = \lambda Id + S$$

  for some scalar λ and some operator $S : C_0(K_{\mathcal{A}}) \to C_0(K_{\mathcal{A}})$ that factors through c_0. In particular

$$\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A)) = \mathbb{R}, \mathbb{C}.$$

- *Whenever the Banach space $C_0(K_{\mathcal{A}})$ is decomposed into a direct sum $C_0(K_{\mathcal{A}}) = \mathcal{X} \oplus \mathcal{Y}$ of two closed, infinite-dimensional subspaces $\mathcal{X}$ and $\mathcal{Y}$, then either $\mathcal{X}$ is isomorphic to $C_0(K_{\mathcal{A}})$ and $\mathcal{Y}$ is isomorphic to c_0, or vice versa.*

Questions:

- What could be $\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A))$ other than $\mathbb{R}, \mathbb{C}$? $C(2^{\mathbb{N}})$? $C(2^{\mathbb{N}}, M_2)$?
- Can one construct in ZFC a maximal $\mathcal{A}$ as above?

The second application

Theorem (P.K., N. Laustsen, 2021)

There is an uncountable, almost disjoint family $\mathcal{A}$ such that:

- *every bounded linear operator $T : C_0(\Psi_{\mathcal{A}}) \to C_0(\Psi_{\mathcal{A}})$ has the form*

$$T = \lambda Id + S$$

  for some scalar λ and some operator $S : C_0(K_{\mathcal{A}}) \to C_0(K_{\mathcal{A}})$ that factors through c_0. In particular

$$\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A)) = \mathbb{R}, \mathbb{C}.$$

- *Whenever the Banach space $C_0(K_{\mathcal{A}})$ is decomposed into a direct sum $C_0(K_{\mathcal{A}}) = \mathcal{X} \oplus \mathcal{Y}$ of two closed, infinite-dimensional subspaces $\mathcal{X}$ and $\mathcal{Y}$, then either $\mathcal{X}$ is isomorphic to $C_0(K_{\mathcal{A}})$ and $\mathcal{Y}$ is isomorphic to c_0, or vice versa.*

Questions:

- What could be $\mathcal{B}(\mathbb{C}_0(\Psi_A))/\mathcal{S}(\mathbb{C}_0(\Psi_A))$ other than $\mathbb{R}, \mathbb{C}$? $C(2^{\mathbb{N}})$? $C(2^{\mathbb{N}}, M_2)$?
- Can one construct in ZFC a maximal $\mathcal{A}$ as above?

Akemann-Donner C*-algebras

Akemann-Donner C*-algebras

- M_2 - 2×2 matrices,

Akemann-Donner C*-algebras

- M_2 - 2×2 matrices,

Akemann-Donner C*-algebras

- M_2 - 2×2 matrices,
- for $\theta \in [0, 2\pi)$ define a 2×2 complex matrix of a rank one projection by

Akemann-Donner C*-algebras

- M_2 - 2×2 matrices,
- for $\theta \in [0, 2\pi)$ define a 2×2 complex matrix of a rank one projection by

Akemann-Donner C*-algebras

- M_2 - 2×2 matrices,
- for $\theta \in [0, 2\pi)$ define a 2×2 complex matrix of a rank one projection by

$$p_\theta = \begin{bmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}.$$

Akemann-Donner C*-algebras

- M_2 - 2×2 matrices,
- for $\theta \in [0, 2\pi)$ define a 2×2 complex matrix of a rank one projection by

$$p_\theta = \begin{bmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}.$$

- For $A \subseteq \mathbb{N}$ define $P_{A,\theta} \in C_0(\Psi_{\mathcal{A}}, M_2)$ by

Akemann-Donner C*-algebras

- M_2 - 2×2 matrices,
- for $\theta \in [0, 2\pi)$ define a 2×2 complex matrix of a rank one projection by

$$p_\theta = \begin{bmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}.$$

- For $A \subseteq \mathbb{N}$ define $P_{A,\theta} \in C_0(\Psi_{\mathcal{A}}, M_2)$ by

Akemann-Donner C*-algebras

- M_2 - 2×2 matrices,
- for $\theta \in [0, 2\pi)$ define a 2×2 complex matrix of a rank one projection by

$$p_\theta = \begin{bmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}.$$

- For $A \subseteq \mathbb{N}$ define $P_{A,\theta} \in C_0(\Psi_{\mathcal{A}}, M_2)$ by

$$P_{A,\theta}(x) = \begin{cases} 0 & x \notin A \cup \{x_A\} \\ p_\theta & x \in A \cup \{x_A\} \end{cases}$$

- M_2 - 2×2 matrices,
- for $\theta \in [0, 2\pi)$ define a 2×2 complex matrix of a rank one projection by

$$p_\theta = \begin{bmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}.$$

- For $A \subseteq \mathbb{N}$ define $P_{A,\theta} \in C_0(\Psi_{\mathcal{A}}, M_2)$ by

$$P_{A,\theta}(x) = \begin{cases} 0 & x \notin A \cup \{x_A\} \\ p_\theta & x \in A \cup \{x_A\} \end{cases}$$

- Let $\mathcal{A} \subseteq \wp(\mathbb{N})$ be an almost disjoint family,

Akemann-Donner C*-algebras

- M_2 - 2×2 matrices,
- for $\theta \in [0, 2\pi)$ define a 2×2 complex matrix of a rank one projection by

$$p_\theta = \begin{bmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}.$$

- For $A \subseteq \mathbb{N}$ define $P_{A,\theta} \in C_0(\Psi_{\mathcal{A}}, M_2)$ by

$$P_{A,\theta}(x) = \begin{cases} 0 & x \notin A \cup \{x_A\} \\ p_\theta & x \in A \cup \{x_A\} \end{cases}$$

- Let $\mathcal{A} \subseteq \wp(\mathbb{N})$ be an almost disjoint family,

Akemann-Donner C*-algebras

- M_2 - 2×2 matrices,
- for $\theta \in [0, 2\pi)$ define a 2×2 complex matrix of a rank one projection by

$$p_\theta = \begin{bmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}.$$

- For $A \subseteq \mathbb{N}$ define $P_{A,\theta} \in C_0(\Psi_{\mathcal{A}}, M_2)$ by

$$P_{A,\theta}(x) = \begin{cases} 0 & x \notin A \cup \{x_A\} \\ p_\theta & x \in A \cup \{x_A\} \end{cases}$$

- Let $\mathcal{A} \subseteq \wp(\mathbb{N})$ be an almost disjoint family,
- Let $\phi : \mathcal{A} \to [0, 2\pi)$ be injective

Akemann-Donner C*-algebras

- M_2 - 2×2 matrices,
- for $\theta \in [0, 2\pi)$ define a 2×2 complex matrix of a rank one projection by

$$p_\theta = \begin{bmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}.$$

- For $A \subseteq \mathbb{N}$ define $P_{A,\theta} \in C_0(\Psi_{\mathcal{A}}, M_2)$ by

$$P_{A,\theta}(x) = \begin{cases} 0 & x \notin A \cup \{x_A\} \\ p_\theta & x \in A \cup \{x_A\} \end{cases}$$

- Let $\mathcal{A} \subseteq \wp(\mathbb{N})$ be an almost disjoint family,
- Let $\phi : \mathcal{A} \to [0, 2\pi)$ be injective
- Akemann-Doner algebra $AD(\mathcal{A}, \phi)$:

Akemann-Donner C*-algebras

- M_2 - 2×2 matrices,
- for $\theta \in [0, 2\pi)$ define a 2×2 complex matrix of a rank one projection by

$$p_\theta = \begin{bmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}.$$

- For $A \subseteq \mathbb{N}$ define $P_{A,\theta} \in C_0(\Psi_{\mathcal{A}}, M_2)$ by

$$P_{A,\theta}(x) = \begin{cases} 0 & x \notin A \cup \{x_A\} \\ p_\theta & x \in A \cup \{x_A\} \end{cases}$$

- Let $\mathcal{A} \subseteq \wp(\mathbb{N})$ be an almost disjoint family,
- Let $\phi : \mathcal{A} \to [0, 2\pi)$ be injective
- Akemann-Doner algebra $AD(\mathcal{A}, \phi)$:

Akemann-Donner C*-algebras

- M_2 - 2×2 matrices,
- for $\theta \in [0, 2\pi)$ define a 2×2 complex matrix of a rank one projection by

$$p_\theta = \begin{bmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}.$$

- For $A \subseteq \mathbb{N}$ define $P_{A,\theta} \in C_0(\Psi_{\mathcal{A}}, M_2)$ by

$$P_{A,\theta}(x) = \begin{cases} 0 & x \notin A \cup \{x_A\} \\ p_\theta & x \in A \cup \{x_A\} \end{cases}$$

- Let $\mathcal{A} \subseteq \wp(\mathbb{N})$ be an almost disjoint family,
- Let $\phi : \mathcal{A} \to [0, 2\pi)$ be injective
- Akemann-Doner algebra $AD(\mathcal{A}, \phi)$: The C*-algebra generated in $C_0(\Psi_{\mathcal{A}}, M_2)$ by

Akemann-Donner C*-algebras

- M_2 - 2×2 matrices,
- for $\theta \in [0, 2\pi)$ define a 2×2 complex matrix of a rank one projection by

$$p_\theta = \begin{bmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}.$$

- For $A \subseteq \mathbb{N}$ define $P_{A,\theta} \in C_0(\Psi_{\mathcal{A}}, M_2)$ by

$$P_{A,\theta}(x) = \begin{cases} 0 & x \notin A \cup \{x_A\} \\ p_\theta & x \in A \cup \{x_A\} \end{cases}$$

- Let $\mathcal{A} \subseteq \wp(\mathbb{N})$ be an almost disjoint family,
- Let $\phi : \mathcal{A} \to [0, 2\pi)$ be injective
- Akemann-Doner algebra $AD(\mathcal{A}, \phi)$: The C*-algebra generated in $C_0(\Psi_{\mathcal{A}}, M_2)$ by

  $c_0(M_2) = \{f : C_0(\Psi_{\mathcal{A}}, M_2) : f|\{x_A : A \in \mathcal{A}\} \equiv 0\}$ and

Akemann-Donner C*-algebras

- M_2 - 2×2 matrices,
- for $\theta \in [0, 2\pi)$ define a 2×2 complex matrix of a rank one projection by

$$p_\theta = \begin{bmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}.$$

- For $A \subseteq \mathbb{N}$ define $P_{A,\theta} \in C_0(\Psi_{\mathcal{A}}, M_2)$ by

$$P_{A,\theta}(x) = \begin{cases} 0 & x \notin A \cup \{x_A\} \\ p_\theta & x \in A \cup \{x_A\} \end{cases}$$

- Let $\mathcal{A} \subseteq \wp(\mathbb{N})$ be an almost disjoint family,
- Let $\phi : \mathcal{A} \to [0, 2\pi)$ be injective
- Akemann-Doner algebra $AD(\mathcal{A}, \phi)$: The C*-algebra generated in $C_0(\Psi_{\mathcal{A}}, M_2)$ by

$$c_0(M_2) = \{f : C_0(\Psi_{\mathcal{A}}, M_2) : f|\{x_A : A \in \mathcal{A}\} \equiv 0\} \text{ and } \{P_{A,\phi(A)} : A \in \mathcal{A}\}.$$

Akemann-Donner C*-algebras

- M_2 - 2×2 matrices,
- for $\theta \in [0, 2\pi)$ define a 2×2 complex matrix of a rank one projection by

$$p_\theta = \begin{bmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}.$$

- For $A \subseteq \mathbb{N}$ define $P_{A,\theta} \in C_0(\Psi_{\mathcal{A}}, M_2)$ by

$$P_{A,\theta}(x) = \begin{cases} 0 & x \notin A \cup \{x_A\} \\ p_\theta & x \in A \cup \{x_A\} \end{cases}$$

- Let $\mathcal{A} \subseteq \wp(\mathbb{N})$ be an almost disjoint family,
- Let $\phi : \mathcal{A} \to [0, 2\pi)$ be injective
- Akemann-Doner algebra $AD(\mathcal{A}, \phi)$: The C*-algebra generated in $C_0(\Psi_{\mathcal{A}}, M_2)$ by

$$c_0(M_2) = \{f : C_0(\Psi_{\mathcal{A}}, M_2) : f|\{x_A : A \in \mathcal{A}\} \equiv 0\} \text{ and } \{P_{A,\phi(A)} : A \in \mathcal{A}\}.$$

Akemann-Donner C*-algebras

- M_2 - 2×2 matrices,
- for $\theta \in [0, 2\pi)$ define a 2×2 complex matrix of a rank one projection by

$$p_\theta = \begin{bmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}.$$

- For $A \subseteq \mathbb{N}$ define $P_{A,\theta} \in C_0(\Psi_{\mathcal{A}}, M_2)$ by

$$P_{A,\theta}(x) = \begin{cases} 0 & x \notin A \cup \{x_A\} \\ p_\theta & x \in A \cup \{x_A\} \end{cases}$$

- Let $\mathcal{A} \subseteq \wp(\mathbb{N})$ be an almost disjoint family,
- Let $\phi : \mathcal{A} \to [0, 2\pi)$ be injective
- Akemann-Doner algebra $AD(\mathcal{A}, \phi)$: The C*-algebra generated in $C_0(\Psi_{\mathcal{A}}, M_2)$ by

$$c_0(M_2) = \{f : C_0(\Psi_{\mathcal{A}}, M_2) : f|\{x_A : A \in \mathcal{A}\} \equiv 0\} \text{ and } \{P_{A,\phi(A)} : A \in \mathcal{A}\}.$$

The existence of big commutative subalgebras in C*-algebras

The existence of big commutative subalgebras in C*-algebras

- (T. Ogasawara; 1954) Every infinite dimensional C*-algebra has an infinite dimensional commutative C*-subalgebra

The existence of big commutative subalgebras in C*-algebras

- (T. Ogasawara; 1954) Every infinite dimensional C*-algebra has an infinite dimensional commutative C*-subalgebra
- (J. Dixmier; 1970) Does every nonseparable C*-algebra has a nonseparable commutative C*-subalgebra?

The existence of big commutative subalgebras in C*-algebras

- (T. Ogasawara; 1954) Every infinite dimensional C*-algebra has an infinite dimensional commutative C*-subalgebra
- (J. Dixmier; 1970) Does every nonseparable C*-algebra has a nonseparable commutative C*-subalgebra?
- (C. Akemann, J. Donner; 1979): CH implies: No. Examples are AF (approximately finite dimensional) of density $\mathfrak{c} = \omega_1$.

The existence of big commutative subalgebras in C*-algebras

- (T. Ogasawara; 1954) Every infinite dimensional C*-algebra has an infinite dimensional commutative C*-subalgebra
- (J. Dixmier; 1970) Does every nonseparable C*-algebra has a nonseparable commutative C*-subalgebra?
- (C. Akemann, J. Donner; 1979): CH implies: No. Examples are AF (approximately finite dimensional) of density $\mathfrak{c} = \omega_1$.
- (S. Popa, 1983) No in ZFC (group C*-algebras of uncountable free groups).

The existence of big commutative subalgebras in C*-algebras

- (T. Ogasawara; 1954) Every infinite dimensional C*-algebra has an infinite dimensional commutative C*-subalgebra
- (J. Dixmier; 1970) Does every nonseparable C*-algebra has a nonseparable commutative C*-subalgebra?
- (C. Akemann, J. Donner; 1979): CH implies: No. Examples are AF (approximately finite dimensional) of density $\mathfrak{c} = \omega_1$.
- (S. Popa, 1983) No in ZFC (group C*-algebras of uncountable free groups).

The existence of big commutative subalgebras in C*-algebras

- (T. Ogasawara; 1954) Every infinite dimensional C*-algebra has an infinite dimensional commutative C*-subalgebra
- (J. Dixmier; 1970) Does every nonseparable C*-algebra has a nonseparable commutative C*-subalgebra?
- (C. Akemann, J. Donner; 1979): CH implies: No. Examples are AF (approximately finite dimensional) of density $\mathfrak{c} = \omega_1$.
- (S. Popa, 1983) No in ZFC (group C*-algebras of uncountable free groups). Examples can be arbitrarily big but far from AF.

The existence of big commutative subalgebras in C*-algebras

- (T. Ogasawara; 1954) Every infinite dimensional C*-algebra has an infinite dimensional commutative C*-subalgebra
- (J. Dixmier; 1970) Does every nonseparable C*-algebra has a nonseparable commutative C*-subalgebra?
- (C. Akemann, J. Donner; 1979): CH implies: No. Examples are AF (approximately finite dimensional) of density $\mathfrak{c} = \omega_1$.
- (S. Popa, 1983) No in ZFC (group C*-algebras of uncountable free groups). Examples can be arbitrarily big but far from AF.
- (T. Bice, P.K.; 2017) No in ZFC. Examples of density ω_1 can be AF: $AD(\mathcal{A}, \phi)$ for $\mathcal{A}$ a Luzin family.

The existence of big commutative subalgebras in C*-algebras

- (T. Ogasawara; 1954) Every infinite dimensional C*-algebra has an infinite dimensional commutative C*-subalgebra
- (J. Dixmier; 1970) Does every nonseparable C*-algebra has a nonseparable commutative C*-subalgebra?
- (C. Akemann, J. Donner; 1979): CH implies: No. Examples are AF (approximately finite dimensional) of density $\mathfrak{c} = \omega_1$.
- (S. Popa, 1983) No in ZFC (group C*-algebras of uncountable free groups). Examples can be arbitrarily big but far from AF.
- (T. Bice, P.K.; 2017) No in ZFC. Examples of density ω_1 can be AF: $AD(\mathcal{A}, \phi)$ for $\mathcal{A}$ a Luzin family.

The existence of big commutative subalgebras in C*-algebras

- (T. Ogasawara; 1954) Every infinite dimensional C*-algebra has an infinite dimensional commutative C*-subalgebra
- (J. Dixmier; 1970) Does every nonseparable C*-algebra has a nonseparable commutative C*-subalgebra?
- (C. Akemann, J. Donner; 1979): CH implies: No. Examples are AF (approximately finite dimensional) of density $\mathfrak{c} = \omega_1$.
- (S. Popa, 1983) No in ZFC (group C*-algebras of uncountable free groups). Examples can be arbitrarily big but far from AF.
- (T. Bice, P.K.; 2017) No in ZFC. Examples of density ω_1 can be AF: $AD(\mathcal{A}, \phi)$ for $\mathcal{A}$ a Luzin family.

Questions:

- Are there in ZFC AF of density $\mathfrak{c}$ with no commutative nonseparable algebras?

The existence of big commutative subalgebras in C*-algebras

- (T. Ogasawara; 1954) Every infinite dimensional C*-algebra has an infinite dimensional commutative C*-subalgebra
- (J. Dixmier; 1970) Does every nonseparable C*-algebra has a nonseparable commutative C*-subalgebra?
- (C. Akemann, J. Donner; 1979): CH implies: No. Examples are AF (approximately finite dimensional) of density $\mathfrak{c} = \omega_1$.
- (S. Popa, 1983) No in ZFC (group C*-algebras of uncountable free groups). Examples can be arbitrarily big but far from AF.
- (T. Bice, P.K.; 2017) No in ZFC. Examples of density ω_1 can be AF: $AD(\mathcal{A}, \phi)$ for $\mathcal{A}$ a Luzin family.

Questions:

- Are there in ZFC AF of density $\mathfrak{c}$ with no commutative nonseparable algebras?
- Is there a bound for densities of AF algebras with no commutative nonseparable algebras?

The existence of big commutative subalgebras in C*-algebras

- (T. Ogasawara; 1954) Every infinite dimensional C*-algebra has an infinite dimensional commutative C*-subalgebra
- (J. Dixmier; 1970) Does every nonseparable C*-algebra has a nonseparable commutative C*-subalgebra?
- (C. Akemann, J. Donner; 1979): CH implies: No. Examples are AF (approximately finite dimensional) of density $\mathfrak{c} = \omega_1$.
- (S. Popa, 1983) No in ZFC (group C*-algebras of uncountable free groups). Examples can be arbitrarily big but far from AF.
- (T. Bice, P.K.; 2017) No in ZFC. Examples of density ω_1 can be AF: $AD(\mathcal{A}, \phi)$ for $\mathcal{A}$ a Luzin family.

Questions:

- Are there in ZFC AF of density $\mathfrak{c}$ with no commutative nonseparable algebras?
- Is there a bound for densities of AF algebras with no commutative nonseparable algebras?

The third application

The third application

Theorem (O. Guzmán, M. Hrušák, P.K., 2021)

The third application

Theorem (O. Guzmán, M. Hrušák, P.K., 2021)

- *(Sacks model) Every Akemann-Doner algebra of density $\mathfrak{c}$ contains a nonseparable commutative C*-subalgebra.*

The third application

Theorem (O. Guzmán, M. Hrušák, P.K., 2021)

- *(Sacks model) Every Akemann-Doner algebra of density $\mathfrak{c}$ contains a nonseparable commutative C*-subalgebra.*

The third application

Theorem (O. Guzmán, M. Hrušák, P.K., 2021)

- *(Sacks model) Every Akemann-Doner algebra of density $\mathfrak{c}$ contains a nonseparable commutative C*-subalgebra.*
- *(Cohen model) There is an Akemann-Doner algebra of density $\mathfrak{c}$ with no nonseparable commutative C*-subalgebra.*

The third application

Theorem (O. Guzmán, M. Hrušák, P.K., 2021)

- *(Sacks model) Every Akemann-Doner algebra of density $\mathfrak{c}$ contains a nonseparable commutative C*-subalgebra.*
- *(Cohen model) There is an Akemann-Doner algebra of density $\mathfrak{c}$ with no nonseparable commutative C*-subalgebra.*

The third application

Theorem (O. Guzmán, M. Hrušák, P.K., 2021)

- *(Sacks model) Every Akemann-Doner algebra of density $\mathfrak{c}$ contains a nonseparable commutative C*-subalgebra.*
- *(Cohen model) There is an Akemann-Doner algebra of density $\mathfrak{c}$ with no nonseparable commutative C*-subalgebra.*
- *(ZFC) There is an Akemann-Doner algebra of density $\mathfrak{c}$ with no density $\mathfrak{c}$ commutative C*-subalgebra.*

The third application

Theorem (O. Guzmán, M. Hrušák, P.K., 2021)

- *(Sacks model) Every Akemann-Doner algebra of density $\mathfrak{c}$ contains a nonseparable commutative C*-subalgebra.*
- *(Cohen model) There is an Akemann-Doner algebra of density $\mathfrak{c}$ with no nonseparable commutative C*-subalgebra.*
- *(ZFC) There is an Akemann-Doner algebra of density $\mathfrak{c}$ with no density $\mathfrak{c}$ commutative C*-subalgebra.*

The third application

Theorem (O. Guzmán, M. Hrušák, P.K., 2021)

- *(Sacks model) Every Akemann-Doner algebra of density $\mathfrak{c}$ contains a nonseparable commutative C*-subalgebra.*
- *(Cohen model) There is an Akemann-Doner algebra of density $\mathfrak{c}$ with no nonseparable commutative C*-subalgebra.*
- *(ZFC) There is an Akemann-Doner algebra of density $\mathfrak{c}$ with no density $\mathfrak{c}$ commutative C*-subalgebra.*

Properties of almost disjoint families involved in the third application

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function)

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
  - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
  - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
  - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
  - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.
- Cohen model

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
  - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.
- Cohen model

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
  - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.
- Cohen model
  - No uncountable almost disjoint family has ω_1-controlled $\mathbb{R}$-embedding property.

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f:\{x_A : A\in\mathcal{A}\}\to\mathbb{R}$ (control function) there is a subfamily $\mathcal{B}\subseteq\mathcal{A}$ of cardinality κ such that $f|\{x_A : A\in\mathcal{B}\}$ extends to a continuous $F:\Psi_{\mathcal{B}}\to\mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
  - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.
- Cohen model
  - No uncountable almost disjoint family has ω_1-controlled $\mathbb{R}$-embedding property.

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
    - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
    - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.
- Cohen model
    - No uncountable almost disjoint family has ω_1-controlled $\mathbb{R}$-embedding property.
    - There is an almost disjoint family of size $\mathfrak{c}$ which contains no uncountable $\mathbb{R}$-embeddable subfamily.

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
  - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.
- Cohen model
  - No uncountable almost disjoint family has ω_1-controlled $\mathbb{R}$-embedding property.
  - There is an almost disjoint family of size $\mathfrak{c}$ which contains no uncountable $\mathbb{R}$-embeddable subfamily.

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
  - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.
- Cohen model
  - No uncountable almost disjoint family has ω_1-controlled $\mathbb{R}$-embedding property.
  - There is an almost disjoint family of size $\mathfrak{c}$ which contains no uncountable $\mathbb{R}$-embeddable subfamily.

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
  - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.
- Cohen model
  - No uncountable almost disjoint family has ω_1-controlled $\mathbb{R}$-embedding property.
  - There is an almost disjoint family of size $\mathfrak{c}$ which contains no uncountable $\mathbb{R}$-embeddable subfamily.
- ZFC

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
  - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.
- Cohen model
  - No uncountable almost disjoint family has ω_1-controlled $\mathbb{R}$-embedding property.
  - There is an almost disjoint family of size $\mathfrak{c}$ which contains no uncountable $\mathbb{R}$-embeddable subfamily.
- ZFC

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
  - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.
- Cohen model
  - No uncountable almost disjoint family has ω_1-controlled $\mathbb{R}$-embedding property.
  - There is an almost disjoint family of size $\mathfrak{c}$ which contains no uncountable $\mathbb{R}$-embeddable subfamily.
- ZFC
  - No almost disjoint family of size $\mathfrak{c}$ has $\mathfrak{c}$-controlled $\mathbb{R}$-embedding property.

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
  - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.
- Cohen model
  - No uncountable almost disjoint family has ω_1-controlled $\mathbb{R}$-embedding property.
  - There is an almost disjoint family of size $\mathfrak{c}$ which contains no uncountable $\mathbb{R}$-embeddable subfamily.
- ZFC
  - No almost disjoint family of size $\mathfrak{c}$ has $\mathfrak{c}$-controlled $\mathbb{R}$-embedding property.

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
  - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.
- Cohen model
  - No uncountable almost disjoint family has ω_1-controlled $\mathbb{R}$-embedding property.
  - There is an almost disjoint family of size $\mathfrak{c}$ which contains no uncountable $\mathbb{R}$-embeddable subfamily.
- ZFC
  - No almost disjoint family of size $\mathfrak{c}$ has $\mathfrak{c}$-controlled $\mathbb{R}$-embedding property.
- Question

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
  - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.
- Cohen model
  - No uncountable almost disjoint family has ω_1-controlled $\mathbb{R}$-embedding property.
  - There is an almost disjoint family of size $\mathfrak{c}$ which contains no uncountable $\mathbb{R}$-embeddable subfamily.
- ZFC
  - No almost disjoint family of size $\mathfrak{c}$ has $\mathfrak{c}$-controlled $\mathbb{R}$-embedding property.
- Question

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
  - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.
- Cohen model
  - No uncountable almost disjoint family has ω_1-controlled $\mathbb{R}$-embedding property.
  - There is an almost disjoint family of size $\mathfrak{c}$ which contains no uncountable $\mathbb{R}$-embeddable subfamily.
- ZFC
  - No almost disjoint family of size $\mathfrak{c}$ has $\mathfrak{c}$-controlled $\mathbb{R}$-embedding property.
- Question
  - Is it consistent that every almost disjoint family of size $\mathfrak{c}$ contains an uncountable Q-family?

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
  - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.
- Cohen model
  - No uncountable almost disjoint family has ω_1-controlled $\mathbb{R}$-embedding property.
  - There is an almost disjoint family of size $\mathfrak{c}$ which contains no uncountable $\mathbb{R}$-embeddable subfamily.
- ZFC
  - No almost disjoint family of size $\mathfrak{c}$ has $\mathfrak{c}$-controlled $\mathbb{R}$-embedding property.
- Question
  - Is it consistent that every almost disjoint family of size $\mathfrak{c}$ contains an uncountable Q-family?

Properties of almost disjoint families involved in the third application

Definition

An almost disjoint family $\mathcal{A}$ has the *κ-controlled $\mathbb{R}$-embedding property* if for every function $f : \{x_A : A \in \mathcal{A}\} \to \mathbb{R}$ (control function) there is a subfamily $\mathcal{B} \subseteq \mathcal{A}$ of cardinality κ such that $f|\{x_A : A \in \mathcal{B}\}$ extends to a continuous $F : \Psi_{\mathcal{B}} \to \mathbb{R}$.

- Sacks model
  - Every almost disjoint family of size $\mathfrak{c}$ has the ω_1-controlled $\mathbb{R}$-embedding property.
  - Every almost disjoint family of size $\mathfrak{c}$ contains an $\mathbb{R}$-embeddable subfamily of size $\mathfrak{c}$.
- Cohen model
  - No uncountable almost disjoint family has ω_1-controlled $\mathbb{R}$-embedding property.
  - There is an almost disjoint family of size $\mathfrak{c}$ which contains no uncountable $\mathbb{R}$-embeddable subfamily.
- ZFC
  - No almost disjoint family of size $\mathfrak{c}$ has $\mathfrak{c}$-controlled $\mathbb{R}$-embedding property.
- Question
  - Is it consistent that every almost disjoint family of size $\mathfrak{c}$ contains an uncountable Q-family?

Main references

Main references

- P. Koszmider, *Banach spaces in which large subsets of spheres concentrate*, `arXiv:2104.05335`

Main references

- P. Koszmider, *Banach spaces in which large subsets of spheres concentrate*, `arXiv:2104.05335`
- P. Koszmider and N. J. Laustsen, *A Banach space induced by an almost disjoint family, admitting only few operators and decompositions*, Adv. Math., 381 (2021), paper no. 107613, 39 pages.

Main references

- P. Koszmider, *Banach spaces in which large subsets of spheres concentrate*, `arXiv:2104.05335`
- P. Koszmider and N. J. Laustsen, *A Banach space induced by an almost disjoint family, admitting only few operators and decompositions*, Adv. Math., 381 (2021), paper no. 107613, 39 pages.
- O. Guzmán, M. Hrušák, P. Koszmider, *On $\mathbb{R}$-embeddability of almost disjoint families and Akemann-Doner C*-algebras*, Fund. Math., 254 (2021), pp. 15–47.

Main references

- P. Koszmider, *Banach spaces in which large subsets of spheres concentrate*, `arXiv:2104.05335`
- P. Koszmider and N. J. Laustsen, *A Banach space induced by an almost disjoint family, admitting only few operators and decompositions*, Adv. Math., 381 (2021), paper no. 107613, 39 pages.
- O. Guzmán, M. Hrušák, P. Koszmider, *On $\mathbb{R}$-embeddability of almost disjoint families and Akemann-Doner C*-algebras*, Fund. Math., 254 (2021), pp. 15–47.

Main references

- P. Koszmider, *Banach spaces in which large subsets of spheres concentrate*, `arXiv:2104.05335`
- P. Koszmider and N. J. Laustsen, *A Banach space induced by an almost disjoint family, admitting only few operators and decompositions*, Adv. Math., 381 (2021), paper no. 107613, 39 pages.
- O. Guzmán, M. Hrušák, P. Koszmider, *On $\mathbb{R}$-embeddability of almost disjoint families and Akemann-Doner C*-algebras*, Fund. Math., 254 (2021), pp. 15–47.